Newtonian Spacetime Structure in Light of the Equivalence Principle

Eleanor Knox
Forthcoming in *The British Journal for the Philosophy of Science*

28.12.12

Abstract

I argue that the best spacetime setting for Newtonian gravitation (NG) is the curved spacetime setting associated with geometrized Newtonian gravitation (GNG). Appreciation of the 'Newtonian equivalence principle' leads us to conclude that the gravitational field in NG itself is a gauge quantity, and that the freely falling frames are naturally identified with inertial frames. In this context, the spacetime structure of NG is represented not by the flat neo-Newtonian connection usually made explicit in formulations, but by the sum of the flat connection and the gravitational field.

Introduction

There is a large literature concerning the best spacetime setting for Newtonian gravitation (NG). There is also a large (and sometimes overlapping) literature examining the benefits of a move to a Newtonian theory in which gravity is geometrized: Newton-Cartan theory or geometrized Newtonian gravitation (GNG). However, the degree to which geometrized Newtonian gravitation in fact provides the answer to the first question has been underappreciated, in part because the full import of the existence of a Newtonian equivalence principle hasn't been acknowledged.[1] I will argue here that GNG is not merely an empirically equivalent, perhaps superior, competitor theory to NG. It is, rather, a formulation that captures the best spacetime setting for Newtonian gravitation, judged by its own lights. In fact, the move

[1]Recent exceptions to this exist: Pooley ([forthcoming]) introduces Newton-Cartan theory in very much the same way that I do here and is much more explicit than most that Neo-Newtonian spacetime fails to capture the full symmetry of Newtonian gravitation; Weatherall ([unpublished]) articulates a definition of theory equivalence on which NG and GNG can be identified; Saunders ([unpublished]) emphasizes a role for a Newtonian equivalence principle.

to a curved spacetime setting for NG is motivated by much the same reasoning that usually impels us to insist that Neo-Newtonian spacetime, and not Newton's absolute space, is the best setting for Newtonian gravity. Neo-Newtonian spacetime, orthodoxy that it may be, actually represents an unstable stepping stone on the path to the correct interpretation of Newtonian gravity.

Some key material I present here is not new; my argument here depends on the symmetry of Newtonian mechanics under linear accelerations, given an appropriate transformation of the gravitational field. This symmetry, articulated by Corollary VI of the *Principia*, is hardly news, nor is the fact that it can drive us to adopt the spacetime structure of geometrized Newtonian gravitation; Friedman, for example, discusses this in his ([1983],pp.95-7).[2] However, although the symmetry is known, its consequences have not been fully incorporated into the spacetime discussion; most classical spacetime discussions focus on neo-Newtonian/Galilean spacetime as the *right, or best* setting for Newtonian theories.[3] Once we acknowledge that the symmetries of neo-Newtonian spacetime are really a special case of the complete group of Newtonian symmetries, it seems that the best interpretation of Newtonian gravitation is as a curved spacetime theory.

Needless to say, neo-Newtonian spacetime plays an important pedagogic and clarificatory role. The conceptual journey from absolute space to curved spacetime structure is made much easier by a pausing at a formalism with a flat affine connection. In some areas the debate is helpfully simplified by appeal to neo-Newtonian structure. But if neo-Newtonian spacetime is merely a helpful idealization or approximation to the real Newtonian spacetime structure, then this should be acknowledged; stepping stones become less helpful when they are mistaken for the riverbank.

It is worth a note on what this paper is not. It is not intended as a faithful historical exercise. There is, of course, no doubt that Newton didn't intend a geometrized reading of his gravitational theory, and I generally agree with John Earman when he comments that:

> It is no doubt fruitless but nonetheless tempting to speculate about whether Newton, had he possessed the relevant mathematics, would have dropped his insistence that the structure of spacetime is immutable because it is the emanative effect of an immutable god in favour of the

[2] Penrose [1968] and Geroch [1978] both argue for the naturalness of the geometrized formulation. See also (Pooley [forthcoming]) for a contemporary discussion.

[3] Of course, in explicit discussions of the symmetries of Newtonian theories, authors are often very careful in their claims. The wide-spread belief that neo-Newtonian spacetime is the natural Newtonian setting is most easily seen in literature that is slightly tangential. Consider, for example, discussions of presentism in Newtonian and special relativistic settings: (Balashov [2000, 2008]; Savitt [2006]).

> idea that the affine structure of spacetime, though not immutable, is determined by laws that are immutable because they are instituted by an immutable god. (Earman [1989], p.35)

That is not to say that there is not much to be gained from historical thought about the issues at hand. Recent work by Simon Saunders ([unpublished]) brings to the fore aspects of the *Principia* that are essential to my arguments here.[4] John Stachel's excellent Newstein fable ([2007]) reminds us that the fact that geometrized gravity was first developed in a relativistic context depended on historical contingencies in mathematics; the notion of an affine connection was developed 'postmaturely'. However, my project is intended in the same spirit in which neo-Newtonian spacetime is usually proposed. The exercise here is to choose the spacetime setting that best matches the symmetries of Newtonian dynamics, regardless of the spacetime structure proposed by Newton.

Here it is again worth commenting on how *not* to conceive of such a project. Philosophers of physics are sometimes tempted to view this kind of interpretational work as something like an exploration of the spatiotemporal metaphysics of a non-actual but possible world in which Newtonian mechanics and gravitation are true. This is a deeply unhelpful way of thinking about the issue! After all, it seems highly unlikely that there *is* a possible world in which Newtonian theory is true, in as much as Newtonian theory can't account for the existence of stable matter. At the very least, any possible world described by Newtonian mechanics doesn't possess the kind of objects whose behavior the theory is meant to capture. We should instead think of the project as one that explores and establishes guidelines for theory interpretation. The study of Newtonian mechanics has, after all, been extremely instructive in understanding the interplay between the symmetries of the dynamics of a theory, and the spacetime it proposes.

Determining the right spacetime setting for a theory is a subtle business. Perhaps the most widely accepted methodological principle for choosing a spacetime is that advocated by Earman ([1989], pp. 45-7): the symmetries of one's spacetime ought to exactly match any universal symmetries of one's dynamics. As Earman puts it, if the symmetries of the dynamics exceed those of the spacetime, we can make do with less spacetime structure, and ought to by Occam's razor.[5] To a large extent, I agree with this. However, once theories involving dynamical spacetime structure are introduced, the standard prescription becomes less straightforward. In

[4] Saunders is also interested in establishing an interpretation of newtonian mechanics that faithfully reflects the wider symmetries of the theory, but his end point is Newton-Huygens spacetime, a close relative of what Earman calls Maxwellian spacetime.

[5] There is of course a second strand to Earman's principle: If the symmetries of the spacetime exceed those of the dynamics, then the dynamical laws cannot be universal, but we won't here consider theories whose spacetime symmetries outstrip those of the dynamics.

the debate between Neo-Newtonian spacetime and the curved, dynamical spacetime of geometrized Newtonian gravitation, it is precisely the dynamics/spacetime division that is at issue. I therefore take our guiding methodological principle to be that one should, wherever possible, avoid postulating physical structure that is underdetermined by the empirical content of the theory. Choosing the best spacetime setting for a theory therefore becomes a matter of avoiding undermotivated, unphysical underdetermination (and of avoiding the indeterminism that so often accompanies it). In the current context, my principle and Earman's will come to the same thing, and both speak in favour of a curved connection for Newtonian gravitation.

I'll begin in section one by reviewing what I take to be the orthodox interpretation of Newtonian gravitation. I'll review the standard moves that are made in endorsing neo-Newtonian spacetime, and treating the gravitational potential as a gauge quantity.

In section two, I'll discuss the non-uniqueness of the gravity/inertia split in NG, and argue that the gravitational field and the flat connection are gauge in much the same way that section 1 took the gravitational potential and structure of absolute space to be.

In section three, I'll look at the role cosmology has played in the debate, and ask whether the symmetries discussed in section 2 are only of concern in an infinite homogeneous universe. I'll argue that we can have reason to be concerned about the gauge nature of the gravitation/inertia split even in a finite universe.

In section four I'll argue that the existence of a Newtonian equivalence principle makes it natural to identify the inertial frames of NG with the freely falling frames. Moreover, Simon Saunders has demonstrated that these are the frames that we (and Newton) are compelled to use in applications of Newtonian gravity in any realistic cosmology. This makes a compelling case for claiming that NG is best interpreted as a theory of curved spacetime.

I'll finish, in section 5, with some brief thoughts on theory equivalence and its relation to the issues discussed here.

1 Newtonian gravity: the orthodox approach

Suppose, with a bit of historical gerrymandering, that our starting point for Newtonian gravitation is a gravitational potential formulation set in absolute space.[6] If

[6]We could, of course, be slightly more historically faithful by starting with a force-formulation of the theory, but considering the move from a force theory to a field theory would take too long here. My formulation here is that of a mid-nineteenth century adherent of absolute space supplied with a textbook in twentieth century differential geometry by a passing time-traveler.

we then borrow contemporary geometrical tools in order to express such a formulation, we take its face-value structure to be represented by the tuple $\langle M, t_{ab}, h^{ab}, \overset{NG}{\nabla^a}$ $, \eta^a, \rho, \phi\rangle$. M here is a four dimensional manifold . This manifold comes equipped with two (degenerate) metrics: t_{ab} is the temporal metric, and h^{ab} is the spatial metric. The two metrics are orthogonal:

$$h^{ab}t_{ab} = 0. \tag{1}$$

$\overset{NG}{\nabla}_a$ is a flat derivative operator on M, compatible with both metrics. η^a is a time-like vector field that serves to identify spatial points at different times, and thus gives the 'rigging' that represents the structure of absolute space. It satisfies

$$\overset{NG}{\nabla}_a \eta^b = 0. \tag{2}$$

ρ is the scalar mass density[7] and ϕ is the gravitational potential, which satisfies Poisson's equation:[8]

$$h^{ab} \overset{NG}{\nabla}_a \overset{NG}{\nabla}_b \phi = 4\pi\rho. \tag{3}$$

Needless to say, such a formulation isn't one you'll find in a contemporary textbook; indeed, I make no claim that anyone has ever defended exactly this formulation. A more orthodox presentation of Newtonian gravity eliminates some of the structure of the above. We'll call such a presentation the 'neo-Newtonian' version of the theory. First, it removes η^a, the 'absolute space field', and second, it replaces the gravitational potential by the gravitational field $-\overset{NG}{\nabla^a}\phi$. Thus a more standard presentation of Newtonian gravity takes it to be well-expressed by the tuple $\langle M, t_{ab}, h^{ab}, \overset{NG}{\nabla}_a, \rho, -\overset{NG}{\nabla^a}\phi\rangle$. This section will look briefly at the two moves required to arrive at this orthodox formulation.

The first of these is takes us from absolute space to neo-Newtonian spacetime. The neo-Newtonian formulation specifies only enough spacetime structure to establish absolute accelerations, and not enough to establish absolute velocities. We make this move because Newtonian gravitation obeys the Principle of Relativity:

[7] If I wished to be slightly more sophisticated, I might point out that the fundamental object here is really some mass-momentum field T^{ab} that determines ρ via $\rho = T^{ab}t_{ab}$. For a more thorough exposition of the technical aspects of Newtonian theories, see e.g.(Malament [2012]).

[8] Henceforth I'll usually omit the explicit spatial metric in favour of raised indices. The reader more familiar with general relativity may wish to note that raising indices with h^{ab} is not quite like raising indices with a full spacetime metric because there is no covariant tensor h_{ab}. Raising indices with h^{ab} is best seen as a prescription for creating tensors whose contravariant indices are purely spatial.

nothing in the internal dynamics of a system distinguishes it from one boosted relative to it. This is, of course, expressed in the *Principia* as Corollary V of the laws of motion:

> COROLLARY V The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.

As a result, Newton's absolute space fails to respect the symmetries of the dynamics, and any piece of structure we introduce to represent it (e.g. η^a) will be very much a 'gauge' quantity. Instead, we introduce a flat connection, $\overset{NG}{\Gamma}$, and associated derivative operator. These are intended to capture the inertial structure of neo-Newtonian spacetime; particles subject to no non-gravitational forces and (per impossibile) isolated from gravitational sources follow geodesics of this connection. Furthermore, the connection naturally defines a class of reference frames, related by Galilean transformations, in which the laws of physics take an invariant form.

The second interpretational move involves the claim that it is the gravitational *field*, $-\overset{NG}{\nabla^a}\phi$, and not the gravitational *potential*, ϕ, that represents real physical structure. This is as it should be: the equations of motion are entirely unaffected by any transformation of ϕ that leaves $-\overset{NG}{\nabla^a}\phi$ unchanged (the trajectory of a massive point particle moving in a gravitational field is given by $\xi^a \overset{NG}{\nabla}_a \xi^b = -\nabla^b\phi$). The gravitational potential is a gauge quantity, and we prefer to eliminate it from our ontology in much the same way that we eliminate other gauge potentials like the electromagnetic potential (modulo worries about the Aharonov-Bohm effect).

This quick summary is, I take it, uncontroversial. It's a standard piece of reasoning that brings us to a form of Newtonian gravitation that more faithfully reflects the symmetries of dynamics. What is more controversial is that the same kind of reasoning, and the same drive to faithfully reflect the symmetries of the dynamics, should lead us to interpret both the gravitational field and the connection as gauge quantities. I'll argue for this over the next two sections.

Before moving on, however, we might want to note even these comments raise questions of theoretical identity: have we moved to a new theory by casting Newton's original theory in this form? I take it that there is a commonly used sense in which the move to a contemporary textbook form of the theory counts as reformulation of the theory that we started with.[9] Needless to say, the strength of the claim that this is a reformulation relies on the gauge considerations discussed here: on the surface, a theory with an absolute space field has a different ontology

[9] See, for example, the discussion in (Jones [1991]).

than one without. But although I'll say more in section 5 about this issue, much of what I wish to establish here can be achieved in the absence of a definition of theory equivalence. Whatever you take the relation between our initial formulation and NG as described above, there is a good case that the relation between NG and GNG is *of the same kind*. That is, exactly the same kind of reasoning that brings us to the above formulation will lead us to a theory with the spacetime structure usually attributed to GNG.

2 Newtonian gravity: additional symmetries

Newtonian gravity set in neo-Newtonian spacetime is designed so as to reflect the theory's symmetries under transformations of inertial reference frame and the gravitational potential. However, it's well known that the full symmetry group of Newtonian gravitation is larger than that of Neo-Newtonian spacetime.[10] For one thing spatially uniform time-dependent accelerations are just as much symmetries of the NG equations of motion as uniform boosts to velocity are; this is enshrined in the *Principia* as Corollary VI of the Laws:

> COROLLARY VI: If bodies moved in any manner among themselves, are urged in the direction of parallel lines by equal accelerative forces, they will all continue to move among themselves, after the same manner as if they had not been urged by those forces.

Nothing in the dynamics uniquely picks out a class of inertial frames corresponding to a flat connection. As long as we insist that the inertial structure is flat, it is radically underdetermined. Moreover, the underdetermination of the flat connection $\overset{NG}{\Gamma}$ looks very like the underdetermination of the timelike vector field η^a that represented the 'rigging' of absolute space.

The freedom in the choice of a flat connection is accompanied by a corresponding freedom in the definition of the gravitational field, precisely because the equivalence of gravitational and inertial mass ensures that a gravitational field can produce universal accelerations. Any transformation $\phi \rightarrow \phi + \psi$, where $\nabla^a \nabla^b \psi = 0$ is a symmetry of the theory, providing that the derivative operator is transformed via $\nabla \rightarrow \nabla'$ where the Christoffel symbol associated with ∇' satisfies ${\Gamma^a}_{bc}{}' = {\Gamma^a}_{bc} - t_{bc} \nabla^a \psi$. These transformations include trivial transformations

[10](Synge [1937]) mentions these additional symmetries. Sklar further expands the informal argument in his ([1976]), and (Friedman [1983],pp.95-7) gives a more formal version. (Pooley [forthcoming]) gives an overview that's very much in line with my presentation here, and (Malament [2012]) gives a more technical presentation.

of the gravitational gauge potential, but also transformations of the gravitational field itself. Not only do the symmetries of the gravitational field and flat connection run parallel to the symmetries of section 1, but the symmetries of section one turn out merely to be a special case of the wider symmetries of the theory.

Of course, it is not enough simply to note that our current formulation involves objects subject to a gauge freedom; as Oliver Pooley notes in his ([forthcoming]), we would also like to know what the gauge invariant objects of the theory are. The above considerations reveal that while the gravitational field and inertial frames are each subject to a gauge freedom, our choice of gauge for the two is not independent. In order to pick out a new class of inertial frames we must choose a new gravitational field. Thus the gauge invariant objects of the theory must be functions of both the gravitational field and the connection; roughly speaking, what is invariant under these gauge transformations is the sum of the gravitational and inertial fields. One such gauge-invariant object,[11] which will here be denoted by $\overset{NG}{I}{}_b{}^a{}_c$, is given by:

$$\overset{NG}{I}{}_b{}^a{}_c = \overset{NG}{\Gamma}{}_b{}^a{}_c + \overset{NG}{\nabla}{}^a \phi t_{bc} \tag{4}$$

The next step in interpreting Newtonian gravitation is to ask what kind of beast this new object is. The answer will hardly come as a surprise to the reader: $\overset{NG}{I}{}_b{}^a{}_c$ is a connection, and not just any connection; it is the curved connection of geometrized Newtonian gravitation, $\overset{GNG}{\Gamma}{}_b{}^a{}_c$.

Thus it turns out that the same kind of reasoning that lead us to Newtonian gravitation set in neo-Newtonian spacetime also leads us to geometrized Newtonian gravitation. Once we have excised the gauge dependent objects, we are left with a theory whose structure is represented by the tuple $\langle M, t_{ab}, h^{ab}, \overset{GNG}{\nabla}_a\ \rho\rangle$. M, t_{ab}, h^{ab} and ρ are defined as in NG above. However, the derivative operator $\overset{GNG}{\nabla}_a$ has curvature, $R^a{}_{bcd}$, which is subject to the following constraints:

$$R_{ab} = 4\pi\rho t_{ab} \tag{5}$$

$$R^{[a}{}_{[b}{}^{c]}{}_{d]} = 0, \tag{6}$$

$$R^{ab}{}_{cd} = 0 \tag{7}$$

where (1) is the now-geometrized Poisson equation. Given these constraints, there is a unique GNG derivative operator and associated connection for any given deriva-

[11]Note that this object is invariant under the discussed transformations of the covariant derivative and gravitational field, but not under general coordinate transformations.

tive operator/gravitational field pair in NG.[12] From the perspective of GNG, equation (4) represents this relationship; rewriting it with the GNG connection replacing our mystery invariant, we get the standard equation for the recovery of the GNG connection from the neo-Newtonian connection and gravitational field:

$$\overset{GNG}{\Gamma}{}_{b}{}^{a}{}_{c} = \overset{NG}{\Gamma}{}_{b}{}^{a}{}_{c} + \overset{NG}{\nabla}{}^{a} \phi t_{bc} \tag{8}$$

But it's not clear here that the move to GNG should be thought of any differently from the move to what I called neo-Newtonian NG above; the motivation for regarding the gravity/inertia split as a mere artefact of our representation seems every bit as good as our arguments for regarding the gravitational potential, or absolute space structure, as artefacts of a representation that should either be excised, or, at the least, not taken seriously as physical objects. We pause at neo-Newtonian spacetime only if we consider a subset of the fullest possible symmetry group of the theory.

However, things are perhaps a little less straightforward than I have presented them to be. While the arguments above are present in the literature, discussions of the gauge status of the gravity/inertia split have historically brought in considerations of cosmology, and it's interesting to see how these might (and might not) play a role here.

3 Cosmological Considerations

Concerns about Newtonian cosmology have their roots in an argument about the potential inconsistency of Newtonian mechanics in the context of an infinite, homogeneous universe. John Norton ([1992]), among others, has argued that, in such a universe, Newtonian mechanics fails to produce predictions for the force on a given body. This is, of course, certainly correct, in as much as the gravitational field, which defines the forces on bodies, is a gauge quantity. As Malament ([1995]) points out (and Norton ([1995]) endorses, at least to some extent), the argument seems to highlight not so much the inconsistency of a certain Newtonian cosmology, but rather the undesirability of taking the gravity/inertia split seriously given the symmetries of the theory.

However, by the lights of the current discussion, this dialectic is somewhat surprising. From the literature alone, a reader might conclude that the alleged inconsistency does not arise if an island universe cosmology is postulated.[13]But nothing in the discussion so far has mentioned cosmology at all. For my purposes here,

[12]That this is the case was first proved in (Trautman [1965]).

[13]This is not to suggest that all parties to the debate believe this. David Malament appears to see the inconsistency as flagging a conclusive problem for an already troubled formulation, one

it's worth turning the argument around; Is it really the case that the gravity/inertia split is *only* troubling in an infinite homogeneous universe? Do island universe boundary conditions really solve the problem?

One reason why one might think that they do is that they appear to suggest a natural fixing of the gauge. If we demand that the gravitational field tends to zero as spatial distance from the center of the island universe tends to infinity, then we naturally adopt a gauge in which the island universe is assumed not to be accelerating. If we accept this, then it seems as if the only case in which the gravity/inertia split is really underdetermined is the homogeneous and infinite one. Our interpretation of NG then seems to rest on the degree to which we demand that the theory be able to deal with any cosmological model, and perhaps even the relative likelihood of island and homogeneous Newtonian cosmologies. How on earth would we evaluate such a thing?

However, the island universe solution to the non-uniqueness of the gravity/inertia split is not quite as secure as is sometimes supposed. After all, the mere availability of some natural fix of gauge doesn't imply that a gauge quantity should be included in our ontology. The naturalness and usefulness of the Lorenz gauge in electromagnetism, for example, does not automatically imply that there is a more fine-grained ontology below the electromagnetic field.[14] Moreover, an island universe seems to give us far too much gauge fixing for free! The proponent of a standard of absolute rest can easily argue that the apparent underdetermination of the time-like vector field that establishes such a standard is broken by the existence of an island universe: in such a universe we can simply take the center of mass to be at rest, and thus fix the velocities. Likewise, we can impose boundary conditions on the gravitational potential that eliminate its gauge freedom. Neither of these arguments are taken to be good arguments for abandoning neo-Newtonian spacetime and taking the potential to be ontologically fundamental.

It's therefore worth making explicit the reasons why we might treat the gravity/inertia split differently. The argument for this rests on the fact that gravitational fields require gravitational sources: in an island universe it is indeed very natural to impose the boundary conditions that we do. However, if part of the question under consideration is whether to take the gravitational field seriously, then this argument is questionable. At best, the proponent of gauge fixing can insist that, from the perspective of their ontology, the gauge fixing is kosher; they can scarcely argue that this fact necessitates the inclusion of the gravitational field in the ontology. But the

---

which demonstrates the advantage of adopting GNG for all applications of the theory, whatever the cosmology.

[14] Note that fixing the Lorenz gauge doesn't quite fix the potential, so this case would never be an argument for taking the potential itself, rather than equivalence classes of potentials, physically seriously.

arguments run deeper that this: as we'll see in the next section, there are pragmatic considerations that may well pull us away from the notion of the inertial structure associated with a flat connection, even in an island universe.

4 A Newtonian equivalence principle: inertial frames in NG

The discussion so far leaves open two questions. First, suppose we grant that the gravity/inertia split does not reflect physical structure, only its 'sum' does. Do we then have reason to identify this 'sum' with a piece of spacetime structure? This question seems particularly pressing when we stay within the formalism of NG, and don't explicitly introduce the invariant object of the theory as a connection with curvature governed by the equations of GNG. However, assuming that not all connections represent spacetime structure[15] we can ask the same question within the formalism of GNG: why think that this connection represents spacetime structure?

A second question was raised already: is there reason to worry about the gravity/inertia split in an island universe? This section will answer both of these via consideration of the nature of an inertial frame within NG. It will argue that the freely falling frames are the most natural candidates for inertial reference frames in NG, even in an island universe. In the Newtonian context, where no spacetime metric exists, the sole role of *spacetime* structure (as opposed to spatial or temporal structure) is to represent the structure of inertial frames. As a result, the connection associated with the inertial frames is the one that represents spacetime structure.

Consider the following form of the equivalence principle (whose name is borrowed from Stachel's ([2007]) fable):

Newstein Equivalence Principle (NEP). *No experiment can distinguish between the effects of a homogeneous gravitational field, and the effects of uniform acceleration.*[16]

As has been often commented, this principle is problematic in the context of general relativity (GR), despite being essential to its development. In a properly geometrized theory, there is no gravitational field proper, and certainly nothing that rightly bears the title of 'homogeneous gravitational field'. However, within the context of NG, this principle is quite enlightening. It expresses exactly the symmetry that leads us to regard the gravity/inertia split as unobservable, and hence

[15]They don't! See (Brown [2005]) for arguments to this effect.

[16]The reader may notice that Newstein's equivalence principle is indistinguishable from what is sometimes called Einstein's equivalence principle.

gauge. Moreover, it's not external to NG, but rather contained within the theory in its earliest form; as Simon Saunders points out, the above principle follows precisely from Corollary VI and the equivalence of gravitational and inertial mass (so long as we assume that 'the motions of bodies among themselves' exhaust the observable phenomena in a Newtonian context).[17]

However, once we've decided, on the basis of the above, to regard only the gravity/inertia 'sum' of equation (4) as meaningful, another problem arises. The NEP implies that a reference frame freely falling in a homogenous gravitational field is indistinguishable from one moving inertially. The natural thing to do now is to follow the reasoning that lead us to assert reference frames related by the Galilean transformations in Newtonian gravity to be equivalent; Einstein's elevator and Galileo's ship have a great deal in common. This would lead us to identify freely falling frames with inertial frames. However, the gravitational fields as defined in NG are not generally homogeneous; as a rule they produce tidal forces that mean that, strictly speaking, real freely falling laboratories are not quite equivalent to those moving inertially far from gravitational fields. In GR we get round this problem by reformulating the equivalence principle to acknowledge that inertial frames are a local matter, where local is a contextually sensitive term.[18] We might therefore wish to introduce the following:

The Newtonian Strong Equivalence Principle (NSEP). *To any required degree of approximation, given a sufficiently small spatial region, it is possible to find a freely falling reference frame with respect to whose associated coordinates the motions of bodies amongst themselves are indistinguishable from those expected in the absence of external forces.*

The NSEP is the principle that could identify locally freely falling frames with inertial frames, and hence establish that the GNG connection is the correct spacetime connection. It is true in both NG and GNG. But here, again, island universe worries might raise their head. In an island universe, gauge fixing for the gravity/inertia split is possible, and it seems that we may have a competing candidate for a class of inertial frames. In such a universe, are there still compelling reasons to identify the freely falling frames with the inertial frames?

It will be helpful here to think more carefully about the operational status of inertial frames in Newtonian gravitation. Saunders ([unpublished]) points out that the neo-Newtonian inertial frames are empirically inaccessible in an island universe to a degree that is sometimes underappreciated. Simply assuming that the

[17] See (Saunders [unpublished, pp.14-5]) for more discussion of the equivalence principle in Newtonian mechanics.

[18] For more details on how to think about this kind of contextual sensitivity in GR, see (Brown [2005], p.170).

center of mass of the universe moves inertially helps us very little with the practical application of the theory. Any system not located at the exact center of such a universe will have an acceleration that depends on distance from the center; if the universe is large, such distant-from-center accelerations will be very large indeed.[19] As a result, one might think it a mystery that Newton could (and believed he could) practically apply his theory. Saunders argues that there is good evidence that Newton was aware of the problem, and went to some lengths to establish that freely falling frames could be treated as inertial for all practical purposes. Roughly speaking, this comes about because the freely falling frames provide convenient non-rotating frames, and Corollary VI then guarantees that such accelerated non-rotating frames may be treated as inertial.

Saunders' aim with the above is to argue that *any* non-rotating frame will suffice for Newtonian mechanics. Paying careful attention to Corollary VI, he argues that the right spacetime setting for Newtonian mechanics is one without the means to determine absolute linear accelerations. From our perspective here Corollary VI is only one half of the symmetry enshrined in the equivalence principle; in order to get at the full symmetry group of the theory, we must consider transformations of the gravitational field as well. But although Saunders and I arrive at different conclusions,[20] parts of his account may also be bent to my purposes here, and used to argue for a curved-spacetime interpretation of NG. For his argument reminds us that, in the context of Newtonian astronomy, for example, the frames we actually use are freely falling. The reference frame of the solar system is precisely the kind of inertial frame picked out by the NSEP; the gravitational influences on the solar system are sufficiently distant that tidal effects are not relevant at the scale of our reference frame. But if these are the reference frames that we actually use, homogeneous infinite universe or no, then the case for adopting curved spacetime is very compelling. To do otherwise at best (in the case of an island universe) commits us to spacetime structure that is practically inaccessible and of no relevance to our physics, and at worst (in the case of an infinite homogeneous universe) commits us to spacetime structure that is inaccessible even in principle.

[19] See (Saunders [unpublished], p.10) for details.

[20] The difference in our accounts, as I see it, rests partly in my focus on the equivalence principle, and partly in the fact that Saunders starts from a force, rather than field, formulation of Newtonian gravitation. The relative merits of these approaches are interesting, but sadly beyond the scope of the present article.

5 Theory Equivalence?

I have tried so far to suppress the question of theory equivalence. However, the reader will have noticed an occasional awkwardness in the language I've used; are NG with a potential in absolute space, NG as a field theory in Newtonian spacetime, and GNG different theories, or reformulations of the same theory? This question might seem intimately connected to the issues at hand; certainly some discussions of the above have come under the heading of theory equivalence. It's therefore worth a brief discussion of our best accounts of theory equivalence and their bearing on this debate. At the same time, I'll put the arguments here into the context of discussions about the epistemology of geometry.

The most established and well worked-out account of theory equivalence is Clark Glymour's ([1970; 1977; 1980]). Glymour holds two theories to be equivalent if they are both empirically equivalent and mutually inter-translatable. For a first order theory Glymour cashes the notion of inter-translatability in terms of definitional equivalence, where two theories are definitionally equivalent just in case there exists a first-order definitional extension of the one theory which is logically equivalent to a definitional extension of the second. [21]

As Glymour points out ([1977]), NG and GNG are not theoretically equivalent on this definition (nor are our two forms of NG). Although the GNG connection is uniquely determined by a given NG connection and gravitational field, the reverse is not true; given some GNG connection, the gravity/inertia split is not unique. And likewise, given some neo-Newtonian flat connection and gravitational field, the absolute space field and gravitational potentials are not uniquely defined. As a result, by Glymour's lights, this paper has dealt with at least three different theories.

In a recent paper, James Weatherall ([unpublished]) suggests an alternative account of theoretical equivalence. Weatherall points out that Glymour's definition appears to give too fine-grained a notion of theory. In particular, it counts as distinct theories which differ only by the presence or absence of a gauge quantity; Weatherall uses formulations of electro-magnetism with and without the vector potential A^μ as an example. He argues (and I agree), that there is a natural sense in which we do not pass to a new theory simply by reformulating in terms of the electromagnetic field $F_{\mu\nu}$. However, A^μ is not uniquely determined by $F_{\mu\nu}$, so by Glymour's definition they are not inter-translatable.

Weatherall proposes an amendment to Glymour's account. Roughly speaking,[22] he holds two theories to be equivalent just in case they are empirically equiv-

[21] I omit many details here because I'll argue that nothing in my argument turns on the details of our definition of theoretical equivalence. For a more thorough account, see Glymour's work, or (Weatherall [unpublished]) for a summary.

[22] Weatherall's complete account involves some category-theoretic apparatus that won't be devel-

alent and for every model of the one theory there is a model of the other theory whose "physical structure" is isomorphic. The *physical* structure of a given model here depends on which models of a single theory we take to be equivalent; the physical structure is just that structure that equivalent models have in common. So if we consider two models related by a gauge transformation to represent the same physical situation, then only the gauge invariant structure counts as physical structure. Weatherall's account can thus allow for the equivalence of two formulations that differ only by the presence or absence of a gauge quantity.

When Weatherall's definition is combined with the arguments in this paper, NG and GNG come out as theoretically equivalent; if both the flat NG connection and the gravitational field are gauge quantities, then it is the gauge invariant 'sum' that comprises the physical structure of NG. And that gauge invariant sum is, of course, identical to some GNG connection for any model of NG.

Weatherall's account fits well with the arguments of this paper. It also fits well with the way in which gauge theories are discussed in both the physics and philosophy literatures. As such, it is a helpful extension of Glymour's original account. But it doesn't, of course, negate the need for the kinds of arguments presented here; NG and GNG only come out as theoretically equivalent once we have established the gauge nature of the NG field and connection. As Weatherall points out, different interpretations of NG lead to different judgements of theoretical equivalence.

So Weatherall's new definition is congenial to the aims of this paper but not, I think, essential. It is very natural, given the arguments here, to think of NG and GNG as reformulations of the same theory, and it is certainly useful to have our theory/formulation language clarified. But if one wishes to adopt Glymour's definition, and insist that these are different theories that differ just by the presence of gauge structure, the conclusions here are not much affected. My aims here are twofold: first, to reiterate the oft-made point that the gravitational field and flat connection of NG are best seen as gauge quantities and, second, to emphasize the less common point that the relationship between NG and GNG is very much like the one between NG with absolute space and a gravitational potential, and neo-Newtonian NG. And of course, the similarity of the relationship carries through on either definition of theory equivalence; on Glymour's definition, the move to neo-Newtonian NG involves a change of theory, as does the move to GNG. On Weatherall's definition combined with the arguments here, all three count as reformulations of the same theory. But either way, there is nothing particularly 'correct' about the neo-Newtonian theory or formulation.

What of the epistemology of geometry? This topic has historically been associated with issues of theory equivalence (see, for example, (Glymour [1977])).

oped here.

Glymour argued against widespread theoretical underdetermination, holding that different theories could have different levels of empirical support despite both being empirically adequate. Applied to the geometrical case, this comes out as an argument against Reichenbach's geometrical conventionalism; according to Glymour, the very fact that GNG has less surplus, or gauge, structure means that it accrues more empirical support than NG. Although these two are different theories by Glymour's lights, they do not comprise a case of underdetermination. Weatherall reaches different conclusions after classing NG and GNG as reformulations of a single theory. He concludes that in the Newtonian context, the question of whether spacetime is curved is not well-posed: because the metric structure of NG admits of both curved and flat connections, spacetime curvature isn't determinate in the way it is in general relativity, and we're free to describe the trajectories of bodies in different ways.

In this area I am more inclined to agree with Glymour than Weatherall or Reichenbach. On the view presented here, the gravitational field and flat connection are pieces of surplus structure. However, a function of these, $\overset{NG}{I}{}_b{}^a{}_c$ in equation (4) is gauge invariant and should be counted as physical structure. And this invariant quantity has all the properties of a curved connection, with all the right links to the rest of physics (via the equivalence principle) to count as a piece of spacetime geometry. As a result NG itself is best interpreted as a curved spacetime theory, albeit written in a form that obscures its geometrical structure. NG and GNG thus possess the same geometry, and are not an example of the underdetermination of spacetime geometry.

6 Conclusions

The reasoning that leads us to regard the gravity/inertia split as not physically meaningful in NG is much the same as the reasoning that leads us to believe that absolute space is not physically meaningful. Moreover, the reasons we have for thinking of the curved GNG connection as representing spacetime structure are of much the same kind (although more persuasive) as those we have for thinking that the flat neo-Newtonian connection represents spacetime; both claim to best capture the inertial structure of the theory. Therefore, in as much as we see the arguments for GNG as persuasive, we should see GNG as expressing the right spacetime structure for NG, in much the same way that neo-Newtonian formulations of the theory are usually supposed to. Neo-Newtonian spacetime is a stepping stone that may, with the appropriate insight, be skipped over.

Funding

The Leverhulme Trust

Acknowledgements

Special thanks are due to Jim Weatherall and Simon Saunders for comments on an earlier draft. Many thanks also to Oliver Pooley and David Wallace for helpful discussion, as well as to audiences in Dubrovnik, Bristol, and at the 2012 PSA in San Diego for interesting feedback. Needless to say, any remaining mistakes are my own.

References

Balashov, Y. [2000]: 'Persistence and space-time: Philosophical lessons of the pole and barn', *Monist*, **83**(3), pp. 321–40.

Balashov, Y. [2008]: 'Persistence and multilocation in spacetime', *Philosophy and Foundations of Physics*, **4**, pp. 59–81.

Brown, H. R. [2005]: *Physical Relativity: Space-time structure from a dynamical perspective*, OUP.

Earman, J. [1989]: *World enough and space-time*, MIT Press Cambridge, Mass.

Friedman, M. [1983]: *Foundations of space-time theories*, Princeton University Press.

Geroch, R.[1978]: *General Relativity from A to B*, University of Chicago Press

Glymour, C. [1970]: 'Theoretical realism and theoretical equivalence', *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, pp. 275–288.

Glymour, C. [1977]: 'The Epistemology of Geometry', *Nous*, **11**, pp. 227–51.

Glymour, C. [1980]: *Theory and Evidence*, Princeton University Press.

Jones, R. [1991]: 'Realism about what?', *Philosophy of Science*, **58**(2), pp. 185–202.

Malament, D. [1995]: 'Is newtonian cosmology really inconsistent?', *Philosophy of Science*, **62**(4), pp. 489–510.

Malament, D. [2012]: *Topics in the Foundations of General Relativity and Newtonian Gravitation Theory*, University of Chicago Press.

Norton, J. [1992]: 'A paradox in newtonian gravitation theory', *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, pp. 412–20.

Norton, J. [1995]: 'The force of newtonian cosmology: Acceleration is relative', *Philosophy of Science*, **62**(4), pp. 511–22.

Penrose, R. [1968]????

Pooley, O. [forthcoming]: 'Substantivalist and relationalist approaches to spacetime', in R. Batterman (ed.), *The Oxford handbook of Philosophy of Physics*, OUP.

Saunders, S. [unpublished]: 'Rethinking Newton's Principia', ⟨users.ox.ac.uk/ lina0174/Acceleration%20v8.pdf⟩.

Savitt, S. [2006]: 'Presentism and eternalism in perspective', *Philosophy and Foundations of Physics*, **1**, pp. 111–27.

Sklar, L. [1976]: 'Inertia, gravitation and metaphysics', *Philosophy of Science*, **43**(1), p. 1.

Stachel, J. [2007]: 'The story of newstein or: Is gravity just another pretty force?', *The Genesis of General Relativity*, pp. 1962–2000.

Synge, J. [1937]: 'On the concept of gravitational force and gauss's theorem in general relativity', *Proc. Edinb. Math. Soc., 2nd Series*, **5**, pp. 93–102.

Trautman, A. [1965]: *Foundations and Current Problems of General Relativity (Notes by Graham Dixon, Petros Florides and Gerald Lemmer)*, Lectures on General Relativity.

Weatherall, J. O. [unpublished]: 'Are newtonian gravitation and geometrized newtonian gravitation theoretically equivalent?'.