Flavour-Oscillation Clocks and the Geometricity of General Relativity

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October 9, 2009

Abstract

I look at the 'flavour-oscillation clocks' proposed by D.V. Ahluwalia, and two arguments of his suggesting that such clocks might behave in a way that threatens the geometricity of general relativity (GR). The first argument states that the behaviour of these clocks in the vicinity of a rotating gravitational source implies a non-geometric element of gravity. I argue that the phenomenon is best seen as an instance of violation of the 'clock hypothesis', and therefore does not threaten the geometrical nature of gravitation. Ahluwalia's second argument, for the 'incompleteness' of general relativity, involves the idea that flavour-oscillation clocks can detect constant gravitational potentials. I argue that the purported 'incompleteness-establishing' result is in fact one that applies to all clocks. It is entirely derivable from GR, does not result in the observability of the potential, and is not at odds with any of GR's foundations.

Contents

Introduction

What makes general relativity a 'geometrical' theory? The answer surely exceeds the scope of a single paper. To give necessary and sufficient conditions for 'geometricity' would at the very least presuppose far greater consensus and clarity concerning the concept of geometry than in fact exists in the literature. Nonetheless, it is certainly possible to highlight aspects of GR that have lead to the popular view that it is a theory of spacetime geometry. Einstein's famous insight, that many of the effects that have traditionally been conceived of as gravitational may in fact be thought of as consequences of acceleration relative to a local inertial frame, is clearly essential to the geometric nature of general relativity. However, despite the uncontroversial nature of this claim, the importance of a clear understanding of this idea is sometimes ignored, obscuring what little clarity exists in the foundations of general relativity.

Ahluwalia ([**?**]) and Ahluwalia and Burgard ([**?**]) have put forward a model for a flavour oscillation clock, providing an interesting investigation of gravitational effects on quantum systems. In two papers (**??**) investigating the behaviour of such clocks in gravitational fields, Ahluwalia has suggested that these clocks challenge the idea that general relativity might provide a good model for gravity in the quantum realm. On the one hand, the failure of the clocks to red-shift in accordance with GR demonstrates a non-geometric aspect of the theory, and on the other hand, their sensitivity to a constant gravitational potential renders general relativity incomplete. I argue that neither of these arguments go through; neither effect is at odds with the foundations of GR. Moreover, there is nothing peculiarly quantum about either effect; both have analogs within the non-quantum domain.

In section 1, I look at Ahluwalia's claim that certain such clocks behave oddly in the presence of a rotating mass; they fail to redshift as general relativity predicts. This leads him to the conclusion that he has discovered a 'non-geometric element' in gravity. I argue that a failure to fully acknowledge that gravitational redshift is a consequence of acceleration leads Ahluwalia to overstate the case. The anomalous redshift of certain flavour-oscillation clocks can be reproduced by accelerating the clock appropriately. This implies that such clocks violate the *Clock Hypothesis*, which states that the ability of a clock to read the proper time along its worldline should be unaffected by its state of motion. While this clock hypothesis violation may be interesting in its own right, it does not itself threaten the geometrical character of gravitation.

I examine Ahluwalia's argument for the 'incompleteness' of a general relativistic description of gravitation in section 2. The suggestion here is that the rate of ticking of a flavour oscillation clock will be influenced by a constant gravitational potential. This implies that the rate of a clock in free-fall in the gravitational field of the earth will fail to match the rate of a clock in flat space at spatial infinity; Ahluwalia claims this to be a direct contradiction of the tenets of GR. I will argue that *any* clock is, in a sense, 'sensitive', to a gravitational potential, but the effect is not measurable and the result is derivable within a GR framework.

Throughout the paper, I will appeal to the Einstein Equivalence Principle. This has been stated in many forms, often with subtly different implications. However, for the purposes of this discussion, I will use the following definition:

EEP. *Locally (i.e. on a scale in which tidal forces are not relevant*[1]*), no experiment can distinguish between the effects of a gravitational field, and the effects of acceleration.*

In the context of GR, the above might seem to presuppose a misleading distinction between inertial and gravitational fields. However, in the context of a discussion concerning possible threats to GR's geometrical status, it is useful to return to the stepping stone which established the identity of gravity and inertia, and reexamine its validity in the light of Ahluwalia's claims.

1 Flavour-Oscillation Clocks and the Clock Hypothesis

1.1 Two types of flavour-oscillation clock

The possibility of a flavour-oscillation clock rests on the fact that relativistic quantum mechanics allows certain particles to exist in a superposition of mass and spin eigenstates. If a system is constructed such that it oscillates between two such superpositions, a kind of quantum clock is produced. Ahluwalia suggests weak-flavour neutrinos and neutral kaons as particularly appropriate for this set-up.

Now consider the following systems.[2] For simplicity's sake, assume that a particle has only two possible mass eigenstates (with eigenvalues m_1 and m_2 where $m_1 \neq m_2$) of spin-$\frac{1}{2}$. Picking some axis, z, along which we shall measure spin projection, we construct one clock such that it oscillates between states where all

[1] The question 'how local is local enough?' is a tricky one, and highly context-dependent. For our purposes here, it suffices to note that nothing in our examples depends on second derivatives of the metric.

[2] The account here is a brief, informal sketch. For details, see (**?**) and (**?**).

spins have the same relative orientations, and a second that oscillates between states which are superpositions of spin up and spin down states.

Clock 1:

Oscillates between states $|Q_a\rangle$ and $|Q_b\rangle$, where:[3]

$$|Q_a\rangle = \frac{1}{\sqrt{2}}|m_1, \uparrow\rangle + \frac{1}{\sqrt{2}}|m_2, \uparrow\rangle, \tag{1}$$

$$|Q_b\rangle = -\frac{1}{\sqrt{2}}|m_1, \uparrow\rangle + \frac{1}{\sqrt{2}}|m_2, \uparrow\rangle. \tag{2}$$

Clock 2:

Oscillates between states $|Q_c\rangle$ and $|Q_d\rangle$, where:

$$|Q_c\rangle = \frac{1}{\sqrt{2}}|m_1, \uparrow\rangle + \frac{1}{\sqrt{2}}|m_2, \downarrow\rangle, \tag{3}$$

$$|Q_d\rangle = -\frac{1}{\sqrt{2}}|m_1, \uparrow\rangle + \frac{1}{\sqrt{2}}|m_2, \downarrow\rangle. \tag{4}$$

In the absence of gravity, the probability of a transition from state $|Q_a\rangle$ to $|Q_b\rangle$ is the same as the probability of a transition from state $|Q_c\rangle$ to $|Q_d\rangle$, and thus the two clocks tick at the same rate.

If we place the clocks at some fixed distance from a non-rotating, weak gravitational source, the two mass eigenstates each pick up a *different* relative phase due to the approximately newtonian potential. This causes both clocks to redshift identically by the amount predicted by general relativity.[4]

However, the situation changes if we consider a situation in which the gravitational source is rotating about the z-axis along which particle spin is to be measured. In this case, the gravitational phases depend not only on mass, but also on spin. In particular, the phase shift experienced by each eigenstate depends on how the spin is oriented relative to the axis of rotation of the source. In the case of Clock 1, the additional phase shift due to the rotation is the same for both halves of each state. Because the transition probabilities depend only on relative, and not absolute, phase shifts, this extra spin related phase has no effect on the rate at which the clock ticks, and Clock 1 continues to redshift according to the predictions of GR.

Clock 2, on the other hand, is oscillating between states involving superpositions of opposite spin states. The two eigenstates involved in states $|Q_c\rangle$ and $|Q_d\rangle$ will therefore pick up equal and opposite phases as a result of the interaction of

[3] I have here set Ahluwalia's 'neutrino mixing angle' equal to $\frac{\pi}{4}$. Nothing in the discussion here depends on this choice.

[4] For details of these relative phases and the effective redshift, see (**?**, pp.1496-1497).

spin with the rotating source.[5] Clock 2, unlike Clock 1, therefore picks up an overall relative phase in this situation. This relative phase does affect the transition probabilities, and hence the ticking rate of Clock 2 is altered. As a result, flavour oscillation clocks of the second variety do not experience the standard gravitational redshift when placed in the vicinity of a rotating gravitational source.

1.2 Quantum mechanics and gravity

Before examining Ahluwalia's diagnosis of the above effect, it is worth making some comment concerning the nature of his predictions. The above discussion (and that of the second half of this paper) compare the behaviour of flavour-oscillation clocks in a gravitational field with general relativistic effects such as gravitational redshift. However, in the absence of a theory of quantum gravity, it might seem mysterious that we can discuss the matter at all. In particular, it seems odd that we can use ordinary non-relativistic quantum mechanics to predict the behaviour of the clock. The model above must therefore be seen as a particularly crude approximation to some real theory which we have yet to develop; it is tempting to assume any odd effects predicted flavour-oscillation clocks are merely a consequence of the flaws of our approximation.

Let us therefore examine the approximation used above. Ahluwalia's rough-and-ready method is in fact a tried-and-tested one: simply work in the weak field limit of GR, in which the first diagonal component of the metric (g_{00}) may be treated as a Newtonian potential. In the case of a non-rotating body, the metric in question is just the Schwarzschild metric, and the potential just the ordinary Newtonian one. In the case of the rotating body, we use the Kerr metric, and the potential picks up a term that depends on the angular momentum of the body. In both cases, quantum predictions are generated simply by incorporating these potentials into the Hamiltonian.

What justifies this method? It is, of course, rather natural, in the absence of a theory of quantum gravity, simply to return to a Newtonian picture, but use GR to generate predicted potentials. On the other hand, its relative naturalness is no guarantee of the accuracy of its results. Nonetheless, the method has worked in some experimentally confirmed cases; for example, the theoretical prediction of the experimentally verified COW effect (**?**) uses just this approximation. Moreover, the method derives some theoretical support from a result due to Rosen (**?**). This

[5] As will shortly be explained, this language is strictly inaccurate in the GR context, where rather than interacting with the source, the clock is affected by changes to the local inertial structure. Throughout my description of these effects, it will occasionally be necessary to adopt Ahluwalia's conceptual framework which treats gravity as a force, rather than a modification of spacetime structure. I hope this will not cause too much confusion.

shows that non-relativistic quantum mechanics is form-invariant under arbitrary accelerative translational motion; accelerating a quantum system has just the same effect as subjecting the system to a potential that affects all parts of the system equally. That is, quantum mechanics obeys the Einstein Equivalence Principle; accelerations and gravitational fields may be modeled in exactly the same manner. If we therefore see the key component of GR as being the insight that a body subject to gravity is accelerating, the inclusion of a potential in the Hamiltonian seems well-motivated.

Inevitably, this is not quite the whole story. Rosen's result does not deal with rotating systems, so the case of the rotating mass may be less straightforward. One might also think that there is something a little odd about using non-relativistic quantum mechanics to model mass superpositions, which are only permitted when we move to the relativistic case.[6] However, one may (as Ahluwalia does) justify the use of the Schrödinger equation by considering the non-relativistic limit of the Dirac equation, and Ahluwalia's approximation has some precedent. Ultimately, the arguments of this paper will in fact lend his model some support, by suggesting that the strange effects he discusses are not peculiar to quantum systems at all.

1.3 A new non-geometric element in gravity?

How are we to interpret the anomalous redshift? Ahluwalia suggests we draw the following conclusions:

> ...the non-geometric element in redshifts may be interpreted as a quantum-mechanically induced fluctuation over a geometric structure of space-time.(**?**, p.1500)

Ahluwalia sees the phenomenon as a quantum fluctuation, because there in fact exists a third variety of flavour-oscillation clock, formed by interchanging the spin states of clock 2. In the presence of a rotating source, this clock will experience a change in rate exactly opposite to that experienced by clock 2. As a result, if we take an equally weighted sample of our three types of clock, the average shift will precisely be the gravitational redshift predicted by GR.[7]

Leaving the issue of fluctuations aside, why claim that these redshifts possess a 'non-geometric' character? For Ahluwalia, this claim depends on the following definition:

[6] See (**?**) for the original result, or (**?**, Appendix 7) for an overview.

[7] Of course this isn't a quantum fluctuation in the usual sense, as there is no uncertainty as to which type of clock some individual system comprises.

Geometry 1. *Geometrical elements are those that are completely specified by the gravitational source. Non-geometrical elements are those that crucially depend on the details of quantum test particles and do not follow from general relativity alone. (**?**, p.1494)*

The idea expressed here is understandable. The universality of gravity is central to its amenability to geometrical interpretations. However, it is far from certain that the above is a useful way to cash out the notion of universality that lies at the heart of general relativity's geometrical nature.

For comparison's sake, let us turn briefly to another effect arising from a combination of gravity and quantum mechanics: the neutron interferometry results from Colella, Overhauser and Werner's famous experiment (**?**). In this setup, the two halves of a neutron beam split in a tilted crystal interferometer pick up a phase difference as a result of the (minutely) varying gravitational potential experienced by each of the two beams. The resulting phase shift turns out to depend on neutron mass. It is possible to view this as a violation of the spirit of general relativity; it has often been claimed in the literature that the effect calls into question the geometrical nature of gravity.[8] Certainly, such a result constitutes a 'non-geometrical element' by Ahluwalia's lights; the magnitude of the effect depends not only on the gravitational force but also on the details of the quantum test particle.

However, there is good evidence that the COW experiment, despite appearances, is not actually at odds with GR. In particular, despite the appearance of a mass term in the phase shift, there need be no suggestion that the experiment violates the Einstein Equivalence Principle. Exactly the same result is predicted if the apparatus is accelerated, a prediction that has been borne out by experiments involving horizontally accelerated interferometers (**?**). It is therefore natural to see the effect as resulting from the acceleration of the interferometer with respect to freely falling inertial frames. This means that it is hard to make precise the notion that the COW experiment demonstrates a 'non-geometrical' effect. In particular, it is hard to see the effect as posing a particular threat to GR; if the appearance of a mass term in an effect caused by acceleration is problematic, it is presumably just as problematic in flat spacetime as it is in curved. It is, moreover, not obvious why mass dependent acceleration terms should pose particular problems for theories concerning spacetime geometry; the COW effect does not, for example, threaten the relativity principle.[9]

It would therefore seem to make sense to ask the same question with regard to Ahluwalia's anomalous redshift: Is it possible to view this effect as purely a result

[8] For example, see (**?**, p.129)

[9] For further details of the COW experiment, and an overview of other neutron interferometry results, see (**?**) and (**?**).

of the acceleration of the clocks relative to local inertial frames? We will see in the next section that the answer is yes.

1.4 Cancelation and Simulation: An alternative account of geometricity.

Chryssomalakos and Sudarsky (**?**) have argued that the behaviour of neutrino-oscillation clocks does not in fact impugn the geometrical nature of gravitation. In the course of their argument, they introduce the following approximate 'phenomenological' criterion for geometricity:

Geometry 2. *Gravity is geometrical if all its effects can be locally canceled (or simulated) by a suitable choice of reference frame in which their description takes place. (**?**, p.607)*

As it stands, this definition is unsatisfactory. It is most certainly not the case that all interesting gravitational effects can be canceled simply by altering the reference frame from which they are described: if Clocks 1 and 2 are comoving, and fail to tick at the same rate, there is no choice of frame which can negate the effect. Moreover, the definition is not restricted in a way that eliminates tidal effects, which are neither simulable nor cancellable. However, Chryssomalakos and Sudarsky are clearly trying to capture the insight that springs from the equivalence principle, that gravitation and inertia are, in the context of GR, two sides of the same coin.

Bearing in mind that, once we move to the full GR picture, the language of simulation and cancelation is not a natural one, we may note that one consequence of the Einstein Equivalence Principle as defined in the previous chapter is that the effects of a homogeneous gravitational field on a system can be simulated, in the absence of gravity, by accelerating it. Another consequence is that a freely falling object should behave as a force-free body would. If we take these two notions instead of Chyssomalakos and Sudarsky's notions of simulation and cancelation in **Geometry 2**, then we appear to have a relevant and pragmatic criterion for the 'geometricity' of a phenomenon:

Geometry 3. *Gravity is geometrical only if its (local, non-tidal) effects disappear entirely when an object is freely falling, and its effects can be simulated by accelerating a system appropriately.*

I have phrased this as a necessary, rather than sufficient condition because the subtleties of general relativity's geometrical character require further examination. Nonetheless, it is interesting and relevant to examine whether Ahluwalia's flavour-oscillation clocks violate **Geometry 3**.

Chryssomalakos and Sudarsky consider the effects of viewing Clock 2 from a local inertial (free-fall) frame. They then note that from this perspective, the difference in the rates of Clock 1 and Clock 2 will persist, but if the clocks are themselves put into free-fall, the effect disappears:

> ...one might conclude that the effect would persist in the freely falling frame. This would be very puzzling to say the least. However, we must be careful and note that if all we do is change the frame of description but not make the experimental apparatus (including the detectors) move with the locally inertial frame, then the above-mentioned situation would ensue. On the other hand, if we make the experimental apparatus (in particular, the detectors) move together with the locally inertial frame, then the effect will indeed disappear as it should. (**?**, p.613)

Thus Chyssomalakos and Sudarsky suggest that the particular gravitational effect in question does disappear in free-fall. As this is the result on which they base their argument for the geometricity of gravity, it is quite clear that the criterion that they have in mind is **Geometry 3** rather than **Geometry 2**. Their concern is not to show that the difference in ticking rate of clocks 1 and 2 may be canceled by moving to an appropriate frame, but rather that the effect Ahluwalia discusses disappears in free-fall.

What of the issue of simulation? Is there an accelerative situation in which we can expect Clocks 1 and 2 to display the same rate difference as they do in the presence of a rotating gravitational source? With some thought, it becomes clear that there is. In GR, it is well-known that a rotating mass 'drags' the local inertial frames in such a way that they are themselves rotating relative to inertial frames at infinity. This is the Lense-Thirring effect, (**?**).[10] It therefore follows that a body held stationary above such a rotating mass is not only accelerating, but also rotating relative to the local inertial frames. This suggests that the effect can be easily simulated: simply put Clocks 1 and 2 into the appropriate accelerated, rotating motion. In such a situation, the spin states of the two clocks would be expected to be affected by their rotational motion in exactly the same way that they are affected by the rotating gravitational source. We therefore have no reason to believe that flavour-oscillation clocks produce any effects at odds with **Geometry 3**.

[10] See also (**?**). For a straightforward overview of gyroscope precession and the construction of an inertial frame based on gyroscopes, see (**?**).

1.5 The Clock Hypothesis

It might be countered that my comments above do not rule out Ahluwalia's conclusion. I have explicitly said that **Geometry 3** is a necessary, rather than sufficient, criterion for geometricity. This certainly leaves logical space for the insistence that Ahluwalia's **Geometry 1** is also a necessary condition for geometricity, and that, by its lights, the flavour-oscillation clock result is 'non-geometrical'.

Nonetheless, I think a clear understanding of the Einstein Equivalence Principle and **Geometry 3** erodes the ground for such a position. For, once we understand that the results of Ahluwalia's thought experiment would be replicated by simply putting the clocks into accelerated, rotational motion, we should, in line with a standard reading of GR, see the clocks in a vicinity of a rotating source as simply *in* such accelerated, rotational motion relative to the local inertial frames. On such a view, Ahluwalia's second variety of flavour-oscillation clock is simply a clock that is affected by certain kinds of motion. That is, we can see it as a clock that fails to obey the *Clock Hypothesis*:[11]

Clock Hypothesis. *The rate of a clock depends only on its instantaneous velocity:*

$$\Delta t = \int_{t_1}^{t_2} (1 - \frac{v^2}{c^2})^{\frac{1}{2}} dt = \int_{t_1}^{t_2} d\tau, \tag{5}$$

where Δt is the time read by the clock, and τ is proper time.[12]

This amounts to asserting that clocks will read off the lengths of their worldlines even when their path is not a geodesic. The validity of the clock hypothesis rests on the idea that a good clock's mechanism should not be influenced by accelerations. For a standard, macroscopic clock, this amounts to something like the assumption that the internal restorative forces which ensure the function of the clock are large compared to the forces that accelerate it. Clearly, while this assumption is reasonable under a wide range of circumstances, it will inevitably fail for certain clocks under certain conditions. Standard mechanical clocks would suffer if we hit them with a hard enough force, and that most classic and reliable of clocks, the pendulum, ceases to work when subjected to any significant acceleration whatsoever.[13]

[11] The clock hypothesis is not often discussed in detail in the literature. A notable exception is (**?**, pp.94-95). For an account of the behaviour of atomic clocks under acceleration, see (**?**, p.172).

[12] I have written the clock hypothesis here as it is usually stated. However, Ahluwalia's predictions for his clocks assume low velocities, and thus the $\frac{v^2}{c^2}$ term above is in fact irrelevant in the current case.

[13] Of course, from the perspective of GR, we should say that a pendulum *only* works when subjected to a constant acceleration.

Under these circumstances, why should we regard flavour-oscillation clocks as particularly inimical to the geometricity of GR? Why is a clock that happens to be affected by rotational motion for well-understood reasons (its ticking rate is intimately connected to spin eigenstates) any more disturbing than a clock that ceases to work when we hit it?[14]

Consider, for example, comparing a rotating type 2 flavour-oscillation clock with a standard, old-fashioned, mechanical pocket watch undergoing extreme linear acceleration. In both cases, we have a clear understanding of why certain types of motion affect the workings of the clock. In the case of the watch, its ticking depends on the ability of its spring-balance to operate. Subject the watch to a force that is large compared to the force exerted by the spring balance, and it will cease to work. In the case of the flavour-oscillation clock, its ticking depends on the relative phase between two spin eigenstates. Subject the clock to a rotational acceleration, and the spin of the two eigenstates will couple to the clock's angular momentum, altering the rate of the clock. The latter case seems no more mysterious than the former. Certainly, it is clear that the problem does not arise as the result of any peculiarities arising from the *quantum* nature of the flavour-oscillation clock; any number of non-quantum clocks would be affected by rotational motion.

In the light of this, it appears that we have two choices. First, we can accept that flavour-oscillation clocks do not, in fact, threaten the geometricity of gravitation, and that **Geometry 1** is not a plausible criterion. Second, we can accept **Geometry 1**, but then accept that the sensitivity of my wrist-watch to accelerations had long-since undermined any possibility of interpreting GR as a geometrical theory.[15] I feel confident that most readers would choose the former.

2 Flavour-Oscillation Clocks in a Constant Potential

In spite of the above, further threats to the geometricity of gravity loom. In a second paper (**?**), Ahluwalia makes an even stronger claim for his flavour-oscillation

[14]Of course, the fact that one case involves rotational and the other linear acceleration could be said to constitute a difference in itself, but not one that is relevant here; both cases involve well-understood failures of the clock hypothesis. Nonetheless, the failure of quantum bodies with spin to behave as they should in General Relativity is interesting, and may occur in other cases. Drummond and Hathrell (**?**) have calculated the effective action in QED with a gravitational field, and found that it predicts deviation from the null geodesics for photons in certain circumstances. In many ways, this effect provides a more serious threat to the geometricity of GR than flavour-oscillations clocks, because the effect involves a violation of minimal coupling. A thorough discussion of the effect may be found in (**?**, pp.165-168).

[15]Of course, Ahluwalia's **Geometry 1** makes explicit reference to a quantum test-particle, but the general thrust is that the behaviour of a clock must not depend on its constitution.

clocks: not only are they held to demonstrate a non-geometric aspect of gravitation, but their behaviour in a constant gravitational potential demonstrates the "incompleteness" of general relativity. According to Ahluwalia, General Relativity requires that a clock in free-fall tick at the same rate as an inertial clock at spatial infinity in an asymptotically flat spacetime. Flavour-oscillation clocks fail to adhere to this, and thus produce a result at odds with the foundations of GR. In section 2.1 I will attempt to reconstruct Ahluwalia's reasoning. I will then argue in section 2.2 that, for a number of reasons, the argument does not go through; the behaviour of flavour-oscillation clocks is not at odds with GR.

2.1 The problem according to Ahluwalia

According to Ahluwalia:

> The conceptual basis of the theory of general relativity asserts that the flat space-time metric...is measured by a freely falling observer on Earth (or, wherever the observer is).(**?**, p.4)

As a result, claims Ahluwalia, a clock in freefall should tick at the same rate as a clock at spatial infinity. However, the rate of a flavour-oscillation clock in free-fall is sensitive to a constant potential, and thus will not be the same as the rate of a clock in flat Minkowski spacetime. Let us see how the argument works:

Consider a flavour oscillation clock of the type introduced in section 1.1. For the purposes of this argument, the spin eigenstates play no significant role, so it is irrelevant whether we consider clocks of type 1 or 2. There are two possible ways of predicting the effects of gravity on a quantum system. One option is to solve the the Schrödinger equation with a gravitational interaction energy term, treating the gravitational potential as a potential like any other:

$$\left[-\left(\frac{\hbar^2}{2m_i}\right)\nabla^2 + m_g V_{grav}(\mathbf{r})\right]\psi(t,\mathbf{r}) = i\hbar\frac{\partial\psi(t,\mathbf{r})}{\partial t}. \tag{6}$$

Alternatively, it is possible to take a route that respects general relativity's geometrical view of gravity. As the work of Stodolsky ([**?**]) has demonstrated, a particle traversing a classical path acquires a phase ϕ that depends on its mass and its proper time:

$$\Phi = \int_A^B m ds. \tag{7}$$

In the weak-field limit for slow moving particles, this becomes:

$$\Phi = \Phi_0 + \phi, \tag{8}$$

where Φ_0 is the phase picked up when no potential is present (at non-relativistic velocities):

$$\Phi_0 = mt, \tag{9}$$

and ϕ is the gravitational part of the phase:

$$\phi = mV_{grav}t. \tag{10}$$

This result accords with the solution to the above Schrödinger equation with a classical potential. By calculating the probability of a transition from state $|Q_a\rangle$ to state $|Q_b\rangle$, we find that the rate of ticking when the potential is zero is given by:

$$\Omega_0 = \frac{(m_2 - m_1)c^2}{2\hbar}. \tag{11}$$

If we now place the clock into some gravitational potential, we find that the ticking rate picks up an additional factor due to the change in relative phase caused by the gravitational phase:

$$\Omega_{V_{grav}} = (1 + V_{grav})\,\Omega_0. \tag{12}$$

Ahluwalia now considers the case of a spherically symmetric, weak field in general relativity, such as that surrounding the earth. In this situation, the metric is given by:

$$ds^2 = (1 + 2V_{grav}(\mathbf{r}))\,dt^2 - (1 - 2V_{grav}(\mathbf{r}))\,d\mathbf{r}^2. \tag{13}$$

A flavour-oscillation clock held stationary above the earth's surface experiences a Newtonian potential $-\frac{GM_E}{r}$. Putting this into equation 12, we get the standard gravitational redshift.

Thus far everything is in accordance with GR, but Ahluwalia has another worry. Equation 12 is not invariant under the addition of a constant gravitational potential. Ahluwalia uses this fact to generate an apparent contradiction with GR. There exists a distant but massive galactic cluster called the Great Attractor. According to Newtonian gravitational theory, the gradient of the potential may be ignored because it drops off as the square of the distance. However, the potential itself has a comparatively large numerical value. For all intents and purposes, Ahluwalia claims, we may consider this potential to be constant. However, a constant potential does not affect the gravitational acceleration of the clock, which depends solely on the gradient of the potential. Because ticking rate is affected by such constant additions to the potential, all neutrino oscillation clocks on earth should be affected by the great attractor. Moreover *even clocks in freefall* will be affected: a constant potential does not influence gravitational acceleration, and therefore is not cancelled when we transform to the freefall frame. When a clock is put into freefall in some potential $V = -\frac{GM_E}{r} + V_{const}$, it should tick at the following rate:

$$\Omega_{freefall} = (1 + V_{const})\,\Omega_0. \tag{14}$$

.

This, Ahluwalia claims, is a direct violation of the foundations of GR, which he takes to imply that a clock in freefall must tick at the same rate as a clock at spatial infinity (i.e., in flat spacetime). Assuming that the potential at spatial infinity is zero, this appears not to be the case. General relativity is, Ahluwalia claims, "incomplete".

2.2 Unpicking the argument

The above argument possesses many strands, and some untangling and reconstruction will be necessary in what follows. Ultimately, even when the problem is put in a more plausible form, we will see that it does not go through.

First, let us turn to Ahluwalia's account of the foundations of GR with which his effect is claimed to be at odds. Recall, there are two related claims here: First, that a freely falling observer should measure the Minkowski metric, and second, that a clock in freefall should tick 'at the same rate' as one at spatial infinity.

The first of these is clearly false. GR does not demand that freely falling observers measure the Minkowski metric. What GR does demand is that, in as much as a freefall observer may be considered to occupy a local inertial frame (i.e. as long as tidal effects may be ignored), she is an inertial observer; that she follows a geodesic, and feels no gravitational forces. The Einstein Equivalence Principle further demands that no *local*[16] observations allow the freefall observer to distinguish between herself and an inertial observer in flat Minkowski spacetime, but this should not be extrapolated to the conclusion that any freefall observer measures the Minkowski metric. *Locally*, any observer, free-fall or otherwise, will measure the Minkowski metric - the structure of GR spacetime is locally Minkowskian - but *globally*, all observers will agree on whether the spacetime they are in is flat or curved.

As for the second, it is not entirely clear what comparing the rate of a freefall clock to a clock at spatial infinity is supposed to represent. It is not possible to directly compare the rates of two distant clocks; we do not peer across space and determine their relative rates. What we may do is compare the rate of one clock to the time coordinates most naturally defined by the other. Operationally, such a procedure corresponds to comparing the time elapsed on one clock with

[16] Here, local simply means on any scale at which tidal effects are not relevant.

the difference in time between two appropriately synchronised clocks at rest with respect to one another, as our first clock whizzes past the other two. This is what we mean when we refer to the 'difference in ticking rate' represented by relativistic time dilation. Is this kind of procedure what Ahluwalia has in mind? If it is, then the claim that a clock in freefall must tick at the same rate as one at infinity cannot be right. Both clocks may, according to GR, be inertial clocks, but, even in flat spacetime, inertial clocks do not tick 'at the same rate' in the above sense. In Minkowski spacetime, one inertial clock will generally experience time dilation relative to the coordinate time defined by two comoving, synchronised, clocks.

Thus, the very foundations of GR with which Ahluwalia claims his results to be at odds turn out not to be foundations of GR at all. Does this close the issue? Not quite; we may still use Ahluwalia's results to derive a problem. In GR, as in Newtonian mechanics, the absolute value of the gravitational potential should not be measurable. This is particularly clear in GR, where gravitational structure *is* inertial structure. The addition of a constant potential clearly does not influence inertial structure, and ought, therefore, have no measurable effects. Another way of seeing this is to note that, in the weak field limit where the concept of a Newtonian potential may be applied, neither the connection coefficients nor the Riemann curvature are affected by the addition of a constant potential to the diagonal terms of the metric; both depend only on derivatives of the metric components and not their absolute values. GR therefore requires that, if there are cases in which it is permissible to add a constant term to the potential in the Newtonian limit, the effect of such a term must not be measurable.

Given the above, we might think that Ahluwalia's result *is* at odds with GR. First, there is claimed to be some physical situation in which a constant gravitational potential ought to be inserted into the metric. Second, it is claimed that such a constant potential will affect the rate of a certain kind of clock. Prima facie, this is at odds with GR, at least if the effect turns out to be measurable.

We will shortly see that this effect will not in fact be measurable. However, let us first turn to what justifies the introduction of a constant potential in the first place. The metric given in equation 13, with $V = -\frac{GM}{r}$, is derived from the Schwarzschild solution to the GR field equations, with the addition of the following assumptions: First, we assume that the field is sufficiently weak that we may apply the linearised form of the metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{15}$$

with $|h_{\mu\nu}| \ll 1$. However, this $h_{\mu\nu}$ is not unique; the form of the above is retained either under global Lorentz transformations, or under infinitesimal coordinate transformations. In order to simplify the field equations, we work in the

Lorentz gauge. That is, we assume the following condition:

$$\bar{h}^{\mu\alpha}{}_{,\alpha} = 0 \tag{16}$$

where

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h. \tag{17}$$

However, this still does not uniquely fix the value of $h_{\mu\nu}$. Ordinarily, we also fix the above such that $h_{\mu\nu}(r = \infty) = 0$, so that our coordinates are Lorentzian far from the source. This ensures that $h_{00} = h_{ii} = 2V$, where V is the Newtonian potential which is also zero at infinity. However, this final condition is not implied by any of the equations of motion, which only fix the gradient of the potential, and not the potential itself. In this case, we may, if we wish, choose not to work with the coordinates imposed above, and adopt some potential $V = -\frac{GM}{r} + V_{const}$, accepting, of course, that our potential will now be non-zero at infinity. This choice of coordinates[17] should, however, have no measurable effects; the equations of motion are invariant under the addition of a constant term to $h_{\mu\nu}$.

The above gives a simple example of a case in which the introduction of a constant potential is allowed. However, it is worth noting that this situation is very different from the one that Ahluwalia describes, despite the fact that he uses a metric that precisely corresponds to the weak field schwarzschild metric expressed in a non-standard gauge:

$$ds^2 = \left(1 - 2(\frac{GM_E}{r} + V_{const})\right) dt^2 - \left(1 + 2(\frac{GM_E}{r} + V_{const})\right) d\mathbf{r}^2. \tag{18}$$

In Ahluwalia's case, V_{const} is the potential generated by the great attractor. However, as Ahluwalia himself acknowledges, this means that the above is strictly inaccurate. The gradient of the potential due to the great attractor is not zero, but merely very small. As a result, the earth, along with any flavour oscillation clocks in its vicinity, will be freely falling towards the great attractor. Just as the earth's gravitational potential disappears when viewed from the rest frame of a clock falling freely towards it, so too will the potential of the great attractor disappear as the flavour oscillation clock falls freely towards it.

However, there are other weak-field situations in which the addition of a constant potential may be appropriate; for example, in the simple spherically symmetric case described above. Alternatively, thinking of the case in Newtonian terms, one might think to introduce a constant potential inside a thin spherical mass shell.

[17] The exact meaning of the alternative coordinate choice will be examined shortly.

As a result, if it were the case that flavour-oscillation clocks could detect the absolute value of the potential, GR would still be in trouble.

So, is the effect predicted by Ahluwalia measurable? If it were the case that *only* flavour oscillation clocks were affected by the addition of a constant potential, it most certainly would be. The ratio of flavour oscillation clocks to other clocks would enable the measurement of the potential.[18] Fortunately for GR, the effect is, in fact, universal. *Any* clock is 'sensitive' to the value of the potential. This was first pointed out by Stodolsky ([**?**]), who considers the possibility of measuring the gravitational potential by means of a clock based on a mass superposition:

> The K^0 clock...slows down when we put it in a region of smaller g_{00}, but then so do all other clocks-the red shift is universal...[W]e are rescued from observing the value of g_{00} locally by a general coordinate transformation which slows down all clocks equally.(**?**, p.395)

We do not need to look at the details of a quantum clock to derive the effect of a potential on ticking rate; it is a consequence of general relativity itself. Consider a clock at rest in the coordinate system in which the weak Schwarzschild metric takes the form given by equation 14. Relative to this frame our clock is at rest, so $d\mathbf{r} = 0$, and we have:

$$ds^2 = \left(1 + 2(\frac{GM_E}{r} + V_{const})\right) dt^2. \tag{19}$$

For small potentials, this gives:

$$\Omega_{V_{const}} = (1 + V_{const}))\, \Omega_{V_{const}=0}, \tag{20}$$

where $\Omega_{V_{const}=0}$ is the rate the clock "would have ticked at", if the constant part of the potential had been zero. The clock appears to be 'affected' by the constant potential. This is just another form of Ahluwalia's result, derived straight from GR without any details of flavour-oscillation clocks at all! It appears that Ahluwalia's effect is in fact universal. However, it is worth noting that while we remain in the non-standard gauge[19] for which the addition of a constant potential has some meaning, there is no region of space for which this potential is zero. The only

[18]It is not clear to what extent Ahluwalia believes that his effect applies to flavour-oscillation clocks alone. At one stage (**?**, p.7), he seems to suggest that all terrestrial clocks will be affected. At another, (**?**, p.3) he implies that only certain quantum systems are sensitive to the potential. At any rate, he fails to see that the effect is both unmeasurable, and, in a certain sense, a straightforward consequence of GR itself.

[19]I here use the term 'standard gauge' to refer to the situation in which we impose the vanishing of the potential at infinity, and non-standard gauge to indicate that this is not the case.

way to make sense of zero potential, and hence of the comparison to $\Omega_{V_{const}=0}$ embodied by the above, is to take $\Omega_{V_{const}=0}$ to be the ticking rate as described in the standard gauge, in which the potential is zero at infinity. There is no way to have a constant potential in some places and zero potential in others while we remain in a single gauge.

This observation leads us into philosophical waters. What should we make of such comparisons of ticking rate across solutions with different boundary conditions? How are the two solutions related? As a result of the diffeomorphism invariance of the full theory, the equations of motion of weak-field GR are invariant under gauge transformations of the weak field potential:

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \xi_{\mu,\nu} + \xi_{\nu,\mu} \tag{21}$$

where ξ are functions small enough to leave the transformed potential small. Our two weak field solutions are related by just such a transformation; we go from one to the other simply by adding a constant term to $h_{\mu\nu}$. We can therefore see our freedom to add a constant potential as a simple instance of the diffeomorphism invariance of GR. What transformation have we imposed? Adding a constant potential everywhere amounts to scaling all time units by a factor of $(1 + V_{const})$, and scaling distance units by a factor of $(1 - V_{const})$. This makes sense of the idea that, relative to the old coordinates, all clocks in our new gauge tick at a different rate. We should therefore insist that the apparent changes caused by the universal addition of a constant potential do not correspond to *physical* changes, but rather to mere redescriptions of the same physical situation.

Of course, this is not to deny that clocks experiencing *different* potentials as expressed in a single gauge may be said to tick at different rates; this is the standard gravitational redshift. However, such comparisons depend only on potential differences, and not on the potential itself.

Thus we see that the effect discussed by Ahluwalia is universal, and we are saved from observing the value of a potential that must, in GR, be unobservable. Moreover, in the realistic case of a constant potential discussed here, the unobservable and unphysical status of the potential has a clear explanation; the addition of a constant potential just corresponds to writing the metric in terms of rescaled coordinates. It seems, therefore, that Ahluwalia's charge of incompleteness does not go through. Certainly, the application of Newtonian concepts to weak-field situations in GR generates conceptual puzzles, but none so deep that they undermine the foundations of the theory.

Conclusion

Neither of Ahluwalia's arguments should be taken to refute general relativity and the geometrical picture of gravity it suggests. The fact that a clock may be constructed that is sensitive to the rotation of a gravitational source need not, in and of itself, threaten the geometricity of gravitation. The result can be interpreted, within the general relativistic context, as being caused by the rotation of the clock relative to the local inertial frames. Certain of Ahluwalia's clocks are sensitive to rotations, and therefore do not obey the clock hypothesis when in rotational motion. This is interesting, but neither unique, nor uniquely quantum; there are other, classical, instances of clock hypothesis violation. The fact that certain systems, which function well as clocks in certain states of motion, fail to measure proper time in all circumstances is not at odds with the foundations of GR.

Similarly, Ahluwalia's second result, concerning the behaviour of flavour oscillation clocks in a constant potential, does not, on close inspection, threaten the geometricity, or the completeness, of GR. In fact the effect discussed does not depend on any feature special to flavour-oscillation clocks, but is rather derivable for any clock, quantum or not. The apparent contradiction of the tenets of GR stems, in part, from a misreading of the consequences of the equivalence principle, proving once more that the importance of clarity concerning the deceptively simple foundations of GR should not be underestimated.

Acknowledgments

This paper arose from discussion with Harvey Brown, who suggested the topic to me, and helped with numerous aspects of the paper. The second half gained a great deal from discussion with David Wallace. I am deeply grateful to both of them. I also owe thanks to Oliver Pooley and Dennis Lehmkuhl for suggestions and helpful discussions.