Effective Spacetime Geometry

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Abstract

I argue that the need to understand spacetime structure as emergent in quantum gravity is less radical and surprising than it might appear. A clear understanding of the link between general relativity's geometrical structures and empirical geometry reveals that this empirical geometry is exactly the kind of thing that could be an effective and emergent matter. Furthermore, any theory with torsion will involve an effective geometry, even though these theories look, at first glance, like theories with straightforward spacetime geometry. As it's highly likely that there will be a role for torsion in quantum gravity, it's also highly likely that any theory of quantum gravity will require us to get to grips with emergent spacetime structure.

Introduction

The phrase 'emergent spacetime' has all the hallmarks of a good buzzword. It sounds radical and disquieting, indicative of contemporary physics' drive to upset our classical world-view. As such, it's exactly the kind of phrase that we should expect to crop up in a theory of quantum gravity; any theory that deserves the title is likely to involve multiple conceptual revolutions. But my aim here is to suggest that, even if counter-intuitive (although that depends on your intuitions!), postulating emergent spacetime is a less radical and surprising move than it might seem. A close look at general relativity (GR) and some variants will demonstrate that the spacetime geometry we experience should be seen as an approximate or effective matter, even in relatively familiar contexts.

For our purposes here, by 'emergent', I'll simply mean 'non-fundamental', and leave to one side questions about what extra content is required for a full definition of 'emergence'.[1] There are two pairs of questions that spring to mind when faced with a theory, or system of theories, that somewhere contains emergent spatiotemporal geometrical structure. The first of these, and the one I'll concern myself with here, is: *Can spacetime geometry be the kind of thing that is emergent? If so, how?* This question crops up whenever our most fundamental theory fails to include any structure that resembles the macroscopic spatiotemporal structure that we probe when we, for example, perform empirical tests of special and general relativity. We must answer this question both in cases where nothing whatsoever resembling a spacetime exists at the fundamental level, and in cases where there is something like spacetime in the fundamental theory, but this has different structure from the spacetime we experience. Another pair of questions one might ask is: *Can we coherently posit a theory that contains nothing like a spacetime at the fundamental level? If so, how?* This latter question is what is under discussion when, for example, Tim Maudlin talks about local beables [27]. It only crops up for theories that do away with spacetime altogether, and it will not be a target of discussion in this paper.

Let us turn back to the first question. Why might one worry that spacetime geometry isn't the *kind of thing* that can be emergent? One common account of the spatiotemporal content of physical theories holds that a theory like GR posits a background or container spacetime, precisely represented by a differentiable manifold and some geometrical objects representing its metric and affine properties. On this view, which I'll call the orthodox view,[2] theories that seem to posit emergent spacetimes seem particularly radical, for no theory that holds spacetime to be non-fundamental can posit spacetime as a background or container for *all* physical processes, nor posit neatly instantiated properties of such an object. The orthodox account can be broken into two claims. The first is the claim that spacetime is a container or

[1]For a suggestion as to what might go into a weak account of emergence see [21]. For an interesting analysis of emergence and examples within physics see [6, 7].

[2]Classic texts like [9] and [12] can be seen as representative of this approach, which Michel Janssen has appropriately called "the church of $\langle M, O_i \rangle$" [20, p.6]. For an extended critique of this kind of approach to spacetime theories, and one which has had a great deal of influence on the ideas presented here, see [2]. But I should be clear here that not all mathematically sophisticated interpreters of GR hold anything like the container approach. It is really the *combination* of the angle brackets approach and a particular metaphysical view of spacetime that is problematic here.

background (often, but perhaps not necessarily, represented by a manifold). The second claim concerns the representational content of mathematical geometrical structure. A theory's ontology is held to be accurately represented by the n-tuple $\langle M, O_i \rangle$, where M is a differentiable manifold, and geometrical fields are among the O_i. These O_i are read off straightforwardly from an appropriate formulation of the theory. Appropriate geometrical objects among the O_i are taken to represent the geometrical properties of spacetime.

The second of these claims ought to be relatively uncontroversial; there is nothing wrong with writing down the formal objects of a theory in angle brackets, nor with asserting that mathematically geometrical objects represent spacetime geometry. However, when embellished with the metaphysical flourish offered by the 'container' view of spacetime, it can cause problems, for it leads to a view on which the connection between spacetime at the geometrical objects that represent it is thought to be more intimate than the usual relationship between physics and mathematics. A container spacetime is for many an inherently geometrical object; something with real and precise geometrical properties, not just properties that can be given a geometrical representation. This leads to a view on which one assumes that certain parts of a geometrical formalism must be *precisely* instantiated by physical structure for a theory to have representational content. But asserting this kind of link obscures the rather subtle ways in which bits of mathematical structure earn their representational stripes. Moreover, if geometry must be *precisely* instantiated, it can't be effectively instantiated only at a certain scale, or in certain domains. It's in this way that the orthodox view is difficult to reconcile with seeing geometrical structure as non-fundamental; it seems to suggest that, whatever spacetime geometry in an emergent spacetime theory might be, the representational content of the mathematical geometry of such a theory must differ substantially from the kind of representational content we're used to.

This, then, will be the view I challenge here. I will argue that, even in general relativity, the link between the metric field and phenomenological geometry is an approximate one.[3] Moreover, some relatively small changes

[3]My approach will depend on a distinction between three kinds of structure: phenomenological structure, fundamental physical structure, and the mathematical structures that might represent one or both of the former structures. The mathematical geometrical structures of a spacetime theory make a claim to represent the geometry of a spacetime, but they're not to be confused with the physical structure of the spacetime theory any more

to the geometrical structure of GR destroy even the approximative link. In Poincaré gauge theories (PGT), where the connection possesses torsion as well as curvature, it is no longer possible for the affine geometry to be reflected in the inertial structure of the theory. Nonetheless, in most case of PGT, there will be effective inertial structure with the usual empirical significance. Thus a more subtle look at the link between the formalism of a theory and the inertial structure it posits reveals that even theories with very straightforward geometrical structure can be thought of as theories positing emergent spacetime.

In what follows I'll hold that our primary empirical access to spacetime geometry comes about via the intermediary of inertial structure; the ways in which we can probe spacetime structure all involve (directly or indirectly) identifying inertial frames and trajectories. Some formal features of inertial frames will also come into play when we look at connections with torsion. Section one will therefore briefly define inertial structure, and comment on its relationship to spacetime geometry.

In section two, I'll discuss the strong equivalence principle, and how to formulate it in such a way that it is both contentful and true in a GR context. I'll argue that these requirements necessitate a context-relative, approximative version of the equivalence principle, and that this makes phenomenological geometry an effective and approximate matter even in GR.

Section three examines a class of theories involving affine connections with torsion as well as curvature: the Poincaré gauge theories. It argues that in such theories the geometry reflected by the inertial structure differs from the explicit geometrical structure of the theory: non-symmetric connections are not associated with inertial frames in the way that symmetric connections are. As a result, even these rather 'un-radical' theories may be thought of as theories with emergent geometrical structure.

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than the mathematics representing any physical entity is to be confused with the entity itself. Distinct mathematical geometries need not represent distinct physical geometries, and we should not unthinkingly assume that every geometry describable by mathematics represents a possible spacetime. So there is a clear distinction between mathematical and physical geometry. What about my reference to phenomenological geometry? Here I mean phenomenology in the physicist's sense: our empirical access to spacetime structure is mediated by a number of distinctly spatiotemporal phenomena - the behaviour of rods, clocks and light rays are some examples. Phenomenological geometry is whatever geometry best describes these empirically accessible phenomena, and again, a claim here will be that it may or may not reflect the fundamental physical structure.

1. Inertial frames

This paper will examine the relationship between the formalism of a theory and the phenomenological spacetime structure associated with it. I'll argue that the spacetime structure that has direct empirical significance doesn't always reflect a theory's precise geometrical structure. Much therefore rides on which phenomena count as endowing spacetime structure with direct empirical significance. My claim here is that identifying phenomenological spacetime structure is a matter of identifying the structure of inertial frames. Our epistemic access to spacetime geometry is provided by identifying inertial trajectories, and identifying reference frames with respect to whose coordinates the non-gravitational physical laws take some simple and invariant form.

It's hardly controversial to claim that spacetime structure and inertial structure are intimately related; a first course in philosophy of physics teaches us to identify neo-Newtonian spacetime structure as that which properly matches the Newtonian structure of inertial frames, and the same story applies in special and general relativity. But my argument here is more concerned with the identification of empirical or phenomenological spacetime geometry, that is, the geometrical structure that is reflected by our measuring instruments, operationalised coordinate systems and the like. One way to get at this kind of structure would be to seek a full operationalisation of the metric and affine structure of a theory like general relativity. I'll argue here that we can access empirical geometry by less challenging means; considering the inertial structure provides a shortcut that allows us to glean the empirical consequences of a theory without going into the messy details of our various measuring devices. Granted, inertial structure is not itself informal, unmathematical, or wholly operationalised, but it provides a useful intermediary between the formal geometrical structure of a theory and its empirical consequences. In this section, I'll first consider how we might give an appropriate definition of inertial structure, and then argue for the relevance of this structure to empirical geometry.

Textbook discussions (including, and perhaps especially, philosophy textbook discussions) of inertial frames often start from their mathematical definition. However, in what follows, I will be interested in relationship between the formal geometrical structure of a theory and the kind of spacetime structure that we empirically access, which depends on inertial structure as defined by the non-geometrical physics of the theory. One of the morals here will be that the explicit geometrical form of a theory and the reference frames picked out by non-geometrical elements can come apart. It's therefore more helpful in this context to begin with a definition that does not already assume that the metric field and affine structure represent the structure of inertial frames; after all, inertial reference frames were relevant in physics long before the development of the geometrical structures that we now associate with them! In Newtonian theories, and in special relativity, inertial frames have at least the following three features:[4]

1. Inertial frames are frames[5] with respect to which force free bodies move with constant velocities.

2. The laws of physics take the same form (a particularly simple one) in all inertial frames.

3. All bodies and physical laws pick out the same equivalence class of inertial frames (universality).

The above combine to form a 'thicker' notion of inertial frame than is sometimes introduced in the literature. Some texts define an inertial frame solely via (1), as one with respect to which a force-free body is represented as moving in a straight line with constant speed. But this kind of definition is not necessarily sufficient to determine inertial structure when faced with rival formulations of a theory; in these situations, we may have to simultaneously determine which bodies count as force free, and what the correct inertial structure is. In both pre-relativistic and relativistic physics, we face this problem when determining whether gravity is indeed a force, or a manifestation of spacetime geometry. Recognizing not just that freely falling bodies

[4]One might, if one wished, take these features to be something like 'constitutive' of our concept of an inertial frame. But I don't mean this to imply that there is something like an inertial frame concept that may be defined once and for all and is common to all spacetime theories. Concepts in physics vary in subtle ways from theory to theory and from application to application (this lesson from Mark Wilson's [38] is well taken). Inertial frames in general relativity do differ from those in, say, special relativity, most notably because they are only defined locally. Nonetheless, it's also highly non-trivial that so many of the features of an inertial frame are retained in general relativity.

[5]For the time being, I'll take 'frame' to be informally defined; we have a good, if imprecise understanding of hte notion of a reference frame in thought experiments like Galileo's ship, or Einstein's elevator. Shortly, I'll be concerned to give a more precise definition of the mathematics we might take to represent a reference frame.

may be considered inertial, but also that the reference frames associated with these bodies bear a special relationship to the rest of physics, allows us to make the move away from considering gravity to be a force. Thus a richer definition of inertial structure can serve as a tie-breaker when underdetermination threatens.[6] Moreover, we need a thick notion of inertial frames if inertial frames are to be instantiated by complex systems, and not just freely falling particles; this connection will be explored further at the end of the section, and will be argued to provide a useful shortcut to understanding the connection between empirically accessible spacetime structure and the metric structure of general relativity.

However, criteria 2 and 3 require some additional comment. Criterion 2, in part, reflects the importance of the principle of relativity in determining inertial frames.[7] However, it goes beyond this by invoking simplicity. Consideration of the Newtonian case reveals that this is necessary; laws of physics can take the same form in classes of accelerating frame, but inertial frames are those frames in which no fictitious forces are present, and thus in which the genuine forces that the theory posits take their canonical form. Criterion three, on the other hand, requires that all laws have the same invariance group, and take their simplest form with respect to the same frame. It's easy to imagine a mathematical setup in which this criterion is violated (although harder to imagine doing physics in such a universe); just imagine that the laws governing one interaction are invariant under Lorentz transformations, and those governing another under Galilean transformations.

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[6]The conventionalism of inertia in Newtonian mechanics has been extensively discussed in the literature. For a modern discussion, see [2, ch.2]. I've argued elsewhere that a sufficiently thick definition of an inertial frame has the power to dissolve certain underdetermination worries in relativistic physics [22].

[7]Roberto Torretti [35, pp.14-20] gives an excellent overview of this kind of way of thinking about inertial frames, detailing Lange's original definition [24] and stressing the centrality of the relativity principle, in the form of Newton's corollary V. One also sees the importance of a simplicity criterion to Torretti's definition in the standard use of Newton's third law to determine inertial structure. It is precisely the application of the third law that singles out inertial frames by preventing the introduction of fictitious forces. As Torretti notes (p.51), when considering inertial frames in special relativity, Einstein eliminated the need for the third law by assuming that light traveled in straight line with respect to an inertial frame, thus assuming prior knowledge of force-free motion. However, one could note that he could equally well have looked for the frames in which Maxwell's equations were valid in their standard form.

In Newtonian and special relativistic physics, the existence of frames satisfying the above definition is hard to deny. In general relativity the existence of inertial frames is historically more controversial; Einstein, after all, held at various points that general relativity was a theory without any privileged reference frames. Beginning with Krestchmann's work [23], there is a long tradition that denies Einstein's conclusions in this area; Oliver Pooley [33, p.18] comments that the view has been largely replaced by one in which general relativity is indeed held to involve inertial reference frames. But the topic remains controversial, and indeed betrays a divide in the philosophical community; Dennis Dieks [8], for example, defends a a kind of general principle of relativity in the GR context, and there are certainly those who think that in the GR context, coordinate systems and reference frames are devoid of physical significance.[8] The details of this debate lie beyond the scope of this paper, and in what follows I'll discuss the role for inertial frames in GR from the perspective of those who are inclined to think that such privileged frames exist.[9]

But those, like me, who hold the view that a GR universe is one with a role for inertial frames must bear in mind two features of GR that make the identification of inertial structure a slightly more complex matter than in previous theories. First, inertial frames are only definable locally, not globally; we'll see in what follows that this requires some careful consideration when formulating the equivalence principle. Second, it is arguable that GR, unlike flat spacetime theories, is simplest in its generally covariant form, and thus does not possess laws that take their simplest form in inertial coordinates. It is therefore necessary to read criterion 2 in such a way that comparisons of simplicity and sameness of form only apply to laws *as referred to some coordinate system*. Again, it's helpful here to remember that this reading is necessary in pre-GR theories as well; in as much as Newtonian physics can be given a generally covariant formulation, one might argue that the laws of physics take the same form in *all* frames. But this is hardly taken to undermine the validity of the Galilean Principle of Relativity!

[8]Jim Weatherall and Erik Curiel have both defended this view to me in correspondence and conversation.

[9]Those in the opposing camp will not agree with all of my conclusions, as I'll lean heavily on the notion of an inertial frame. But they might perhaps at least accept my final point, that the geometry of Poincaré gauge theories bears a different relationship to operationalised geometry than one might assume.

The existence of inertial frames in general relativity ultimately depends on the validity of the equivalence principle, which will be the topic of the next section. However, before we move on to this, it is worth looking at a geometrical definition of a reference frame, and of a 'normal' reference frame adapted to a particular connection. Although these geometrical objects don't *automatically* match up with the definition of an inertial frame above, we'll see in the next section that the role of the strong equivalence principle in GR is precisely to ensure that, at least locally, normal frames associated with the metric field do indeed play the role defined above.[10]

In order to give a mathematical description of an inertial frame with the right features to be applicable in general relativity, let's begin with a tetrad field $\{E_i\}$ on some neighbourhood $N \subseteq M$ of a manifold M. Although tetrads are often defined to be orthonormal, for the time being we'll work with a broader definition of a tetrad as any set of basis vectors for the tangent space. The next thing to note is that certain tetrad fields (but not all) are associated with coordinates on N: these associated coordinates are the $\{x^i\}$ such that $\{E_i = \frac{\partial}{\partial x^i}|_N\}$. A tetrad field is *holonomic* on a neighbourhood just in case it's possible to define associated coordinates everywhere on the neighbourhood. Physical reference frames can be assigned coordinates, so we'll require that a tetrad field be holonomic in order to represent a physical reference frame.

But of course, *inertial* frames require further conditions. If we are interested in the inertial frames naturally associated with some pseudo-Riemannian metric $g_{\mu\nu}$ with connection $\Gamma^{\rho}{}_{\mu\nu}$ (which need not, in the general case, be the Levi-Civita connection), then we're interested in what I'll call normal, orthonormal frames.[11] Call a frame *normal* on N just in case:[12]

1. The connection coefficients vanish with respect to the frame $\{E_i\}$ on N: $\Gamma^i{}_{jk}|_N = 0$.[13]

[10] This role for the strong equivalence principle has been particularly championed by Brown [2].

[11] This terminology is confusing! Note that not all normal frames are orthonormal, and vice versa.

[12] The discussion here follows [19, Ch1-2].

[13] In discussions of GR, where the connection is the Levi-Civita connection, this requirement is sometimes replaced by the requirement that the first derivatives of the metric vanish, which is sufficient for the Levi-Civita connection to vanish. Note also that where connection coefficients vanish, coordinate axes will be parallel transported along geodesics

Call a frame *orthonormal* on U just in case:

3. The metric takes the form of the Minkowski metric with respect to the frame on N: $g_{ij}|_N = \eta_{ij}$ where $\eta = diag(-1, 1, 1, 1)$.

Coordinates $\{x_i\}$ can be called normal/orthonormal everywhere on N just in case their associated frame $\{\frac{\partial}{\partial x^i}\}$ is normal/orthonormal everywhere on N. But we must be careful with terminology here! As we'll shortly see, what are often called Fermi normal coordinates are not generally normal everywhere in a neighbourhood.

Armed with this definition, we can give a straightforward definition of an inertial frame in special relativity: in Minkowski spacetime, where the connection is the flat Levi-Civita connection, normal orthonormal coordinates may be defined everywhere on the manifold. The associated tetrad field provides a global reference frame in which the flat Levi-Civita connection vanishes and the metric takes the standard diagonalized Minkowski form. This mathematical object has a good claim to represent a special relativistic inertial frame.

In general relativity, of course, things require some adjustment. In the presence of curvature, inertial frames are only locally, not globally defined, so as we might expect there are no global, holonomic, normal and orthonormal tetrad fields definable on any curved neighbourhood. However, if we consider the properties of the tetrad field on some not necessarily open subset U of the neighbourhood N, we find a notion of inertial reference frame that is a candidate for representing a *local* inertial frame. For in a GR spacetime we can if we wish define a holonomic tetrad field on N that is normal and orthonormal along a given *curve*, although it won't generally be normal or orthonormal elsewhere in N off the curve. I'll call such this a locally normal frame. The coordinates associated with such a tetrad field are *Fermi coordinates*,[14] normal and orthonormal along the curve, but generally not elsewhere in the neighbourhood. (Note that, confusingly in the current context, these are often called Fermi *normal* coordinates, although they're not normal everywhere on N.)

But inertial frames in GR should not be associated with just any curve: our local inertial frames should be the freely falling frames; the spatial origin

of the connection, another condition sometimes imposed on inertial coordinates.

[14]For more on Fermi coordinates, see e.g.[26], [19, pp.84-93] or [29].

of the coordinates of such a frame should trace a geodesic path. In order to ensure this, let's restrict our attention to those holonomic tetrad fields (and associated Fermi coordinates) that are normal and orthonormal on a geodesic. Such tetrad fields now have the right features locally to represent inertial frames, inasmuch as they approximate the features of tetrad fields representing global inertial frames within a small neighbourhood of the geodesic.

In general relativity it's the job of the strong equivalence principle to ensure that the locally normal frames and coordinates defined with respect to the metric do indeed link to the rest of our physics in the right way to ensure that tetrad fields with the right features locally play the role of inertial frames in our theory. However, we might be worried that there's still a relevant link missing. The arguments that follow will depend on it being the case that establishing the effective or emergent nature of inertial structure within a domain also establishes the effective or emergent nature of phenomenological geometry within a domain. As mentioned earlier, we need to be sure that inertial structure is an appropriate middle man when trading in operationalized geometry. This in turn relies on the idea that fixing inertial structure will fix all the relevant operational features of physical geometry. It also requires that there be no alternative way of operationalizing spacetime geometry that's independent of inertial structure.

There are at least two reasons why one might be worried about this. First, we might be concerned that inertial structure only reflects affine structure, and doesn't serve to fix distances and hence metric structure. Happily, in a relativistic context, that's not the case; if we know the full set of timelike geodesics (the inertial trajectories) then this is sufficient to fix both conformal and projective structure. This in turn fixes metric structure up to a global scale factor (not just a conformal factor), and thus determines all relative distances, which I take it is sufficient to fix the operational content of the metric.[15]

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[15]This result dates at least back to [10]. One might worry here that my definition of spacetime structure is *too* rich to do the job here; this result only requires knowledge of the free motions, rather than the full inertial frame structure as defined here. Indeed, one might worry that frames fitting the 'thick' definition may not exist in regions of spacetime that are not well-behaved (near black holes for example). I don't intend here for the fulfilment of the thick definition of inertial frame everywhere to be a necessary condition for our empirical access to spacetime. Rather, the definition of an inertial frame here

A second worry might be that discussing an abstract structure like inertial structure is unhelpful if we're interested in phenomenological physical geometry. Typically, discussions of operationalizing the metric center[16] around much more direct means of surveying it, for example with rods and clocks, or with light rays and freely falling particles. If we're interested in looking at instantiated geometry in GR and other theories, why go via the middle man of inertial structure? Why not just look at how free particles, light rays, rods and clocks behave in the theory, and examine whether they *exactly* survey the metric? These (with the exception of free particles, which we don't often come across, and taking 'rigid experimental setups' to count as rods) are the kinds of things that we in fact use when determining spacetime structure.

The answer to this depends on the proposed operationalization/surveying technique. It's certainly relatively straightforward to 'construct' the metric using the set of light ray and free particle trajectories, because these trajectories give us the affine and conformal structure[17]. However, in practice, using this kind of method leaves us open to worries about conventionalism; for this to be a viable technique, we must know already which particles count as free. In the absence of an awareness of 'thick' inertial structure (which in turn depends on the form of the non-gravitational interactions), our definition of free motion will be underdetermined.

Why not simply analyse the behaviour of rods and clocks then? What counts as a rigid body, or a good clock, is much less a matter of convention than what counts as a free particle, in part because these systems are more common and can be compared, and in part because their complexity ties them very firmly to the details of our whole physics. However, it's exactly their complexity that makes it hard to mount arguments about physical geometry in GR based on 'constructive' premises; it's hardly practical to try to model the full class of rigid bodies and clocks from the bottom up. In special relativity, we can adopt the constructive relativist's shortcut, a lá Harvey Brown and Oliver Pooley [4, 5], and note that the Lorentz covariance of the laws governing rods and clocks ensure that they survey the Minkowski

should fix our interpretation of the theory, thus allowing us to determine the force-free trajectories. Once determined, we can acknowledge inertial trajectories in regions where our full notion of inertial structure breaks down.

[16]For example in [4],[5], [2] and [10].

[17]Note that in fact we could make do with just the particle trajectories in defining this structure; if we know all timelike geodesics, we can derive the null geodesics

metric. However, as Brown points out [2, ch.9], the situation is much more complicated in GR, where only local symmetries exist. In order to understand rigid bodies and clocks here, we must turn to the strong equivalence principle, which ensures that the laws governing physical systems are such that the structure of the GR metric is reflected by their behaviour. However, once we've appealed to the strong equivalence principle we've appealed to exactly the information that is encoded in the inertial structure. On lesson of Brown's constructive relativity is that, rather than being an abstract middle man, inertial structure, where it exists, turns out to be exactly the structure that our operational systems can probe.

Other methods by which we might probe spacetime or survey the metric likewise depend implicitly on an awareness of inertial structure. For example, one might think that devices that directly measure tidal forces, and hence curvature, provide our best chance of directly accessing spacetime structure. However, we must build such devices in a way that accounts for inertial effects; the workings of such devices will be affected by accelerations.[18] It's this dependence of other measuring devices on knowledge of inertial structure (in this case, knowledge of acceleration relative to inertial frames) that leads me to claim that our epistemic access to spacetime structure is mediated via our knowledge of inertial frames.

2. General relativity and the effective equivalence principle

In order to determine that frames normal and orthonormal along a geodesic are indeed inertial frames, we need to ensure that the frames picked out by the connection are the same frames in which force free bodies move with constant velocities, and other laws take their simplest form. As pointed out above, the strong equivalence principle claims to do just this. However, the existence of a contentful equivalence principle has been contested ever since J.L. Synge's famous comment:

[18] One can sometimes build a device in such a way that certain kinds of acceleration do not affect its operation - for example, the gravitational gradiometer discussed by Misner, Thorne and Wheeler [28, pp. 401-402] is built in such a way as to be unaffected by linear accelerations, but depends for its operation on being in a particular rotational state. I know of no such device that is immune to all accelerations, and the very need to construct the device in a way that compensates for inertial effects implies awareness of a role for inertia in our physics.

> The Principle of Equivalence performed the essential office of midwife at the birth of general relativity, but I suggest that the midwife be now buried with appropriate honors... [34]

Is scepticism with respect to the usefulness of the equivalence principle justified? I can think of two primary reasons to think so. First, it proves rather a delicate matter to state the equivalence principle in such a way that it is both contentful and true in general relativity; many early formulations fall into triviality or falsehood. Second, one might be puzzled as to what such a principle is meant to be; it is intended to act as some kind of constraint on our theory? If so, why should we accept it? Or is it an empirical principle, akin to the light postulate in special relativity, in which case why isn't it simply superseded by the full theory in all its glory?[19]

Writing down an equivalence principle that is both contentful and potentially true will be the topic of much of this section; I hope the proof of the pudding will be in the eating. But the second question deserves some comment; the role of the equivalence principle has not been as thoroughly explored in the literature as one might hope.[20] To my mind, the equivalence principle is not so much a constraint on the form of our matter theories as the expression of a fact about those theories, that, if it holds true, is remarkable. Furthermore, it expresses just that fact about our matter theories that must be true if systems formed from appropriate matter are to reflect the structure of the metric field, that is, if phenomenological geometry is to reflect the geometry of the metric field. (This last, of course, is one of the moral's of Harvey Brown's work, and it is one that even those not sympathetic to the anti-substantivalist agenda of constructive relativity might absorb). So it is not so much that the strong equivalence principle, if true, would add

[19]There is perhaps a third worry arising from a general scepticism about the existence of a fundamental role for coordinate systems and reference frames in general relativity; inasmuch as the equivalence principle involves these ideas, it can't be fundamental. This scepticism springs from a viewpoint on which reference frames have no role to play in general relativity. Such scepticism will not be directly addressed here; as noted earlier, it reflects a philosophical division that goes beyond the scope of this paper. But we might wish to note that worries about the equivalence principle are often used to motivate the view. Insofar as this section provides a defense of the principle, one might also think it provides a defense of a view on which inertial frames are central to our interpretation of GR.

[20]The work of Harvey Brown [2, 3] and Michael Friedman [13] stand as notable exceptions to this

anything to a complete description of matter, but rather that it provides a concise expression of the properties of matter fields relevant to the task at hand.

2.1. Pauli's 1921 strong equivalence principle

In order to navigate the debates about exactly how to express the principle, let's start with its earliest clear formulation, Pauli's 1921 version:

> For every infinitely small world region (i.e. a world region which is so small that the space and time variation of gravity can be neglected in it), there always exists a coordinate system $K_0(X_1, X_2, X_3, X_4)$ in which gravitation has no influence either on the motion of particles or on any other physical processes. [32, p.145]

The objections to this formulation are well-known. It start with there's the awkward reference to 'gravity' and 'gravitation'. In GR, it's by no means clear what is meant by the gravitational field and there are several candidates: among them the metric field, the curvature and the connection. Synge [34] objected to Pauli's version on the grounds that it is natural to identify a gravitational field with curvature, which is a tensorial quantity, and therefore can't be eliminated by a coordinate transformation. In as much as any suggestion along these lines is merely an attempt to attach a pre-relativistic label to a theory in which the concept has no natural extension, it is not clear that this is a substantive debate. No object in GR is neatly analogous to a Newtonian gravitational field; there's an important sense in which gravity is *eliminated* in GR and replaced with inertial structure.

There's also a whiff of tautology to Pauli's formulation; it's perilously close to "gravity can be ignored in regions in which gravity can be ignored". And yet it's relatively straightforward to understand the spirit of the suggestion: the idea here is to insist that *locally* we may think of freely falling reference frames as inertial frames, and thus of frames that are 'stationary' in a gravitational field as accelerating. We need to find a way of restating this idea *without* reference to gravity, and in such a way that our definition of 'local' is non-circular.

Ultimately I'll recommend a formulation that isn't far off a standard textbook version of the principle. However, in order to understand the consequences of my formulation, it's instructive to look at two alternative classes of formulation, both of which diverge importantly from Pauli's.

2.2. The geometrical SEP

One way of making the equivalence principle more precise is to appeal directly to the geometrical structure of GR. Thus, Trautman [36] suggests that we take the principle to state that all possible experiments must determine the same affine connection. The appeal to experiment here sounds pleasingly empirical; Trautman lists a number of experiments that operationalise the notion of parallel transport. However, it's not clear that these experiments, in and of themselves, fully capture the intended content of something like Pauli's 1921 equivalence principle. Ohanian [30] thinks that Trautman successfully captures an important feature of the equivalence principle, that gravity is universal, but fails to capture the idea that the laws of nature must take an appropriate form in the free-fall frames. This seems right given Trautman's focus on parallel transport; defining parallel transport does not by itself constrain the symmetry group of the laws.

It might be argued that, whatever Trautman intended, we can ensure that something like his SEP has content by analysing what it means for some physical system to determine an affine connection. This is of course correct, but now the work is being done by this supplemental story! As I've presented it here the job of the SEP is precisely to provide a link between non-gravitational systems and the metric and connection; the whole point of the SEP is that it's meant to tell you what it *means* for experiments to pick out a connection.

The same kind of problem attaches to any account that puts too much weight on the geometrical structure of GR. So, for example, one sometimes hears the equivalence principle stated as the idea that spacetime is locally flat, or locally Minkowskian. What exactly should this be taken to mean? As Harvey Brown has pointed out [2, p/170], if it means that the tangent space at each point of the manifold is Minkowskian, it simply states a mathematical fact about pseudo-Riemannian manifolds, without any connection to phenomenology. Absent an account of why the rest of physics behaves, at a local level, as if it's in flat spacetime, references to local flatness again fail to provide a link between mathematical and physical structure.

2.3. The pointy SEP

Another quite widespread kind of equivalence principle formulation shrinks the domain of validity of the SEP to a single spatial point. This is a natural response to the worry that tidal effects never *really* vanish at a particular scale; no extended region of a curved spacetime is actually flat. Such a 'pointy' equivalence principle was originally suggested by Ohanian [30] and has been taken up since by, for example, Ghins and Budden [14].[21] As it turns out, the motivation behind a pointy formulation is not quite right, so we may not, after all, be forced to make sense of the rather unphysical notion of a reference frame at a point.

First, the motivation: Ohanian claims, and others have agreed, that any extended version of the SEP can't be right because certain systems that are sensitive to tidal effects remain equally sensitive on the smallest scales. Ohanian considers a body subject to nothing but tidal forces, and suggests that, because the tidal forces are of the same order as the dimension of the system, the shape deformation remains the same as the scale becomes smaller. Obviously, no coordinate transformation will eliminate such an effect. Therefore Ohanian suggests that there can be no justification for assuming that gravitational effects can be transformed away in infinitesimally small regions.

However, Ohanian's example is an artificial one because he deliberately neglects the restorative forces that give the body its shape. As Ohanian himself acknowledges, in any realistic case, tidal forces will scale down relative to other forces. His example depends on a fictional case with sufficiently low surface tension in the drop, but, no matter how small the surface tension, it will eventually become large in comparison to tidal forces as the system becomes smaller.[22] Ignoring restorative forces in this case is not an acceptable idealization; the restorative forces aren't negligible here, and what we're interested in when formulating the SEP *is* just how the non-gravitational interactions come to be formulated in such a way that real physical systems survey the metric.

[21]Ghins and Budden, while agreeing with Ohanian on a pointy formulation, raise an additional worry, expressing concern regarding which laws should come under the scope of the SEP. The result is what they call the 'punctual equivalence principle', which states that at every point a reference frame may be found in which the "fundamental dynamical and curvature-free special relativistic laws"[14, p.43] hold in their standard form. However, such a formulation robs the SEP of much of its content. Budden and Ghins' formulation will be vacuously filled if *none* of the non-gravitational fundamental laws turn out to be curvature free, for example if all interactions couple to curvature - a situation ordinarily thought of as a straightforward example of SEP violation.

[22]I owe this point to David Wallace (in conversation).

2.4. The effective strong equivalence principle

With the geometrical and pointy avenues closed to us, it seems that something like Pauli's version of the SEP is the most promising. But of course we need to change it so that mention of gravitation is eliminated, and so that the reference frames it appeals to are defined over an extended region in a non-circular way. I propose the following slightly toyed with textbook version:

Strong Equivalence Principle (SEP). *To any required degree of approximation, given a sufficiently small region of spacetime, it is possible to find a reference frame with respect to whose associated coordinates the metric field takes Minkowskian form, and the connection and its derivatives do not appear in any of the fundamental field equations of matter.*

This formulation most certainly has content; it represents a highly non-trivial constraint on the form of the non-gravitational interactions and ensures the existence of physical freely falling inertial frames in GR. The immediate consequences of this SEP may be divided into two parts. First, the principle as stated ensures the universality of the reference frames in question, because the principle requires that at any point the *same* set of reference frames reduces all the fundamental laws to their gravitation-free form. In a theory that fulfills the SEP, there is no scope for one interaction to pick out one set of frames, and another interaction another. Second, the SEP implies the minimal coupling requirement. Minimal coupling is often expressed as the 'comma goes to semi-colon' rule in GR, whereby the GR form of an equation is achieved by replacing all derivatives with covariant derivatives. Alternatively, one can insist that only the connection, and not its derivatives (or, equivalently, only first derivatives of the metric) appear in the fundamental equations. Given that the components of the connection disappear in the freely falling frames, this requirement automatically returns gravitation free equations in freefall.[23]

However, the phrase *"to any required degree of approximation, given a sufficiently small region of spacetime"* requires some explanation. What does

[23] Despite its common usage, the comma goes to semi-colon rule is not in fact a unique prescription, and does not itself guarantee equations that are first order in the metric. The second description of minimal coupling above should therefore be considered more fundamental. See, for example, [37, pp. 374-375] or [2, pp.170-172].

it mean to find a reference frame with respect to which e.g. the connection vanishes approximately? How local is local enough for our Fermi coordinates? That depends on what systems and what kind of physics we're interested in, and on what apparatus we have available.[24] Our measuring devices will, at best, operationalize the not-quite-normal-on-most-of-the-neighbourhood Fermi coordinates, but for small enough neighbourhoods (hopefully larger than the apparatus) they will behave exactly as if they were operationalising normal coordinates in flat spacetime.[25]

Needless to say, this kind of approximative and contextual equivalence principle does not provide a precise link between metric structure and our operationalised inertial frames. The phenomenological correlates of the theory's geometrical structure are always subject to a kind of coarse graining that we don't see in the theory's geometry. This kind of phenomenological coarse-graining goes deeper than the simple fact that operationalizing any piece of mathematical structure requires some approximation; it's a matter of principle, not mere practice, that operationalised reference frames are objects of finite spatial extent and therefore can't perfectly instantiate metric structure. Put another way, the link between metric structure and spacetime as experienced and experimented in by us is 'fuzzy'.[26]

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[24]See [2, p.170], where Brown also stresses the context-dependence of the notion of locality.

[25]Jim Weatherall has suggested to me the following way of making the notion of approximation here precise: Consider a manifold-metric pair $\langle M, g\rangle$ and some geodesic curve γ in the manifold (the curve in question needn't be a geodesic for the proof, but we're concerned with Fermi coordinates in the neighbourhood of a geodesic). Around every point on γ there exists a neighbourhood N on which one can find a flat metric η such that η and its Levi-Civita connection agree with g and its Levi-Civita connection on γ. Fermi coordinates are those whose associated tetrad field is normal and orthonormal everywhere on N relative to the flat metric η and its connection (and hence normal and orthnormal relative to g on γ). We can now define a neighbourhood that meets our approximation standard by demanding that g and η be sufficiently close to another within that neighbourhood. To do this pick some positive definite metric h and demand that η and g be within ϵ of one another relative to h: $Max_N(h^{am}h^{bn}(g_{ab} - \eta_{ab})(g_{mn} - \eta_{mn})) \leq \epsilon$.

[26]Although the analogy shouldn't be pushed too hard, the reference here to Albert and Loewer's 'fuzzy link' as applied to GRW theory is deliberate. We're accustomed to questions about the connection between the basic ontology of quantum mechanics and ordinary language talk being tricky, but it's sometimes assumed that in classical theories the connection is neat and straightforward. Much can be gained by broadly applying the same kind of semantic subtlety that we're forced into in the quantum case!

It's interesting to note that the motivations for the above form of the equivalence principle are entirely internal to GR; they don't require us to consider alternatives to GR, or to consider theoretical scenarios in which GR turns out itself to be an effective or emergent theory. Nonetheless, the picture here is not one of a phenomenological geometry that perfectly correlates with the manifold and metric field, but rather one of a phenomenological geometry whose relationship to the metric is provided by a contextual and approximate principle. On this view, phenomenological geometry is exactly the kind of thing that *could* be emergent; nothing about our direct experience of spacetime structure, or the role it plays in the rest of our physics, demands that we think of spacetime as a background manifold with exact geometrical properties.

This is not to suggest that GR itself postulates emergent spacetime. GR in isolation, thought of as a fundamental theory, postulates the metric field as one of its fundamental objects; just because the way in which the metric represents empirical geometry is less straightforward than one might assume doesn't make geometry emergent. But now suppose, as will likely be required by quantum gravity, that we come to understand GR itself as an effective or non-fundamental theory. Does it make sense to think of the strong equivalence principle as holding in such a situation?

Several modifications are required in order to introduce the strong equivalence principle in an emergent theory. For one thing, as stated, the SEP refers to 'the fundamental field equations of matter'. For another, the principle as written assumes that we can always consider as low a degree of approximation and hence as small a local region as we like; this won't be the case if we are discussing theories that only apply at some minimum scale.

These two problems are relatively easy to deal with. If GR is emergent, then the SEP certainly won't hold with respect to the *fundamental* equations. However, if we know, given some scale or domain, which non-gravitational equations apply, it's straightforward to restrict the SEP so that it refers to only the relevant equations. So, for example, if our theory of quantum gravity predicts violations of GR and classical electromagnetism only on small scales, or in certain domains, the SEP might still hold true of electromagnetism in appropriate scales and domains. Likewise, we need to ensure that the reference frames to which the principle refers are of an appropriate size to the theoretical regime in question, where by 'theoretical regime' I mean whatever class of theories holds with respect to the domain or scale under consideration. Let us therefore introduce a modified equivalence principle:

Effective Strong Equivalence Principle (ESEP). *The ESEP holds relative to some theoretical regime just in case, to any degree of approximation appropriate to the regime, given a sufficiently small region of spacetime, it is possible to find a reference frame with respect to whose associated coordinates the metric field takes Minkowskian form, and the connection and its derivatives do not appear in any of the equations of matter relevant to the regime.*

This principle is likely to fail in certain pathological domains. For example, in regions where the radius of curvature of the effective metric is small compared to the plank length, there will be no reference frame appropriate to the macroscopic scale on which tidal effects may be ignored. But such pathological regimes are just where we might expect our effective theory to break down! In better behaved regions the principle can still apply, and provide the same the *kind* of connection the normal SEP provides between the metric and the geometry we experience.

3. Physical geometry in the presence of torsion

The above suggests that *emergent* GR could be thought of as an emergent spacetime theory without doing too much violence to the foundations of the theory. However, it doesn't provide us with a worked out example of emergent spacetime. To do that we'd need both a theory of quantum gravity and an understanding of how GR could be recovered from such a theory; this is a task for another day! But it will be instructive to look at a scenario in which inertial structure is an effective matter, and does not reflect the explicit geometry of the fundamental theory; my contention here is that Poincaré gauge theories give us just such an example, and thus provide an example of emergent spacetimes in a theoretical package that's superficially surprisingly similar to GR.

Poincaré gauge theories (PGTs) are, if not themselves theories of quantum gravity, interesting precursors to such theories.[27] They are motivated by the desire to cast gravity in a gauge form resembling the gauge theories of the standard model. Although there's a wide class of theories that come under the PGT umbrella, they are united both by their gauge form, and by the

[27] In particular, they bear a relation to string theory; see [15] for a brief explanation of this relationship.

introduction of an affine connection with torsion, as well as curvature. In most cases, this torsion is associated with spin, and thus arises when we consider coupling of gravity to the Dirac equation. Usually, torsion of the connection is argued for via the need to introduce a connection and metric with sufficient independence such that spinors may be represented.[28]

3.1. The geometry of Poincaré gauge theories

Let's start by discussing explicit geometrical structure, and for the time being ignore the details of the gauge structure of these theories. Interesting though it is that gravity may be given an explicitly 'gauge' form (and not just in the attenuated sense in which GR is sometimes said to be a gauge theory of the diffeomorphism group), the argument here only involves understanding some geometrical structure, and considering the possible ways in which that geometrical structure can arise in a PGT Lagrangian.

Mathematically, we can think of a 'spacetime'[29] as a manifold endowed with a metric, $g_{\mu\nu}$ and an affine connection ${\Gamma^\rho}_{\mu\nu}$. In GR, there is a very close relationship between the connection and the metric: a unique Levi-Civita connection $\{^{\rho}_{\mu\nu}\}$ may be derived from the metric by imposing two conditions:

1. The connection must be metric compatible; i.e. the metric must be covariantly constant with respect to the connection (this ensures that the angle between two vectors remains the same under parallel transport):

$$\nabla_\rho g_{\mu\nu} = 0 \tag{1}$$

2. The connection must be symmetric in its lower indices:

$$\{^{\rho}_{\mu\nu}\} = \{^{\rho}_{\nu\mu}\} \tag{2}$$

For every metric, there is exactly one connection, the Levi-Civita connection, that satisfies these two constraints. The Levi-Civita connection is the only connection whose geodesics (defined by parallel transport) match

[28]For a brief summary of this motivation, and an excellent short introduction to PGT, see [18].For a slightly more lengthy introduction, see [31] or [1].

[29]Scare quotes because I don't think these *mathematical* spacetimes are all candidates for representing physical spacetime.

the geodesics of the metric (defined as the curves that (locally) maximize the interval between two points).

However, mathematically speaking, we can also consistently describe a wider class of spacetimes by relaxing the conditions above. If we demand only metric compatibility of the connection, and not symmetry, we get the class of Riemann-Cartan 'spacetimes'. These have connections with torsion, where the torsion tensor measures the asymmetry of the connection when written in a coordinate basis:

$$T^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\mu\nu} - \Gamma^{\rho}{}_{\nu\mu} \tag{3}$$

Because the difference between two connections is always a tensor, any non-symmetric connection can always be decomposed into a symmetric connection and a tensor that includes the antisymmetric part. Of particular interest is the relationship between a general connection and the Levi-Civita connection, which are connected by the contorsion tensor, $K^{\rho}{}_{\mu\nu}$:

$$\Gamma^{\rho}{}_{\mu\nu} = \left\{ {\rho \atop \mu\nu} \right\} + K^{\rho}{}_{\mu\nu} \tag{4}$$

$$K^{\rho}{}_{\mu\nu} = \frac{1}{2}\left[T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu}\right] \tag{5}$$

If $\Gamma^{\rho}{}_{\mu\nu}$ is a metric-compatible connection with torsion, $\left\{ {\rho \atop \mu\nu} \right\}$ won't be equal to the symmetric part of $\Gamma^{\rho}{}_{\mu\nu}$ (the contorsion tensor isn't the torsion tensor, and thus is not the antisymmetric part of $\Gamma^{\rho}{}_{\mu\nu}$). The geodesics defined by $\Gamma^{\rho}{}_{\mu\nu}$ depend only on its symmetric part, and will differ from those defined by $\left\{ {\rho \atop \mu\nu} \right\}$. For a given metric, only its Levi-Civita connection will have geodesics that match those of the metric; where the connection has torsion, metric and affine geodesics will not coincide. Thus Riemann-Cartan spacetimes have two independent sets of geodesic paths.

3.2. Torsion and inertial frames

Poincaré gauge theories all involve connections with torsion. We can therefore immediately see that deriving any PGT's inertial structure directly from its geometrical structure can't be straightforward. We have two classes of geodesic; nothing in the geometry tells us which, if either of these, is associated with inertial structure.

In fact, things are more complicated than this. For it turns out that, although non-symmetric connections do indeed have a set of geodesics associated with them (defined in the usual way: the connection parallel transports their tangent vectors), they *don't* have an associated class of mathematical reference frames that are candidates for representing physical inertial frames. This is because no *holonomic* tetrad field can be normal with respect to a connection with torsion even at a single point. As a result, there is nothing like Fermi coordinates associated with such a connection. A simple thought that might lead to this conclusion comes when we note that the torsion tensor is, as the name suggests, a tensor, and therefore the antisymmetric part of a given connection simply *can't* be made to vanish by an appropriate choice of coordinates. As a result, a non-symmetric connection can't even satisfy the mathematical constraints listed for inertial frames in section 1 (let alone satisfy the physical definition).

Until some relatively recent literature, the story would have ended with the above observation. However, a 1995 paper by David Hartley [16] argues that it *is* possible to choose a reference frame in which a non-symmetric connection vanishes. As a result, it's worth going into a little more technical detail here.

Recall section 1's discussion of the mathematical definition of a reference frame. In this, I said that inertial reference frames in some neighbourhood N of a geodesic were represented by *holonomic* tetrad fields $\{\frac{\partial}{\partial x^i}\}$ that were normal along that geodesic. The requirement that they be holonomic allowed them to be associated with some coordinate system $\{x^i\}$ that is also normal along the geodesic. There turn out to be a number of results relevant to the existence or otherwise of normal coordinates.[30] It can, as expected by the above reasoning regarding the torsion tensor, be shown that[31] there are no Fermi coordinates (that is, coordinates for which the connection coefficients vanish along some curve) for connections with torsion. However we may nonetheless be able to find a *frame*, or tetrad field on N that is normal along a curve, or indeed over the whole of N. Such frames, however, will invariably be *anholonomic* if the connection has torsion; their basis vectors will not commute. As a result, there will be no coordinates associated with such a frame on N.[32]

With this in mind, let's look at Hartley's 1995 result: Given some arbitrary tetrad $\{E_i\}_p$ at a point p, we can extend it over some neighbourhood

[30]These results are detailed and generalised in [19], and I'll follow the results of this text.

[31]See [19, p.40]

[32]See [19, pp.41-41].

$N \subset M$ in such a way that $\nabla E_i = 0$ at p. Hartley shows that for any connection $\Gamma^{\rho}{}_{\mu\nu}$ (with or without torsion), there always exist coordinates $\{x_i\}$ and a frame $\{X_i\}$ on a neighborhood of p such that *at* p:

$$E_i = \frac{\partial}{\partial x^i} \tag{6}$$

and

$$\Gamma^i{}_{jk} = 0 \tag{7}$$

where roman indices indicate that the connection is being expressed in the basis $\{E_i\}$.

How is this compatible with the fact that (as Hartley acknowledges), there are no coordinates in which a connection with torsion vanishes? As noted above, for connections with torsion, the frame in which the connection vanishes isn't a coordinate frame; there are no coordinates associated to it over the region N. Even though the basis $\{E_i\}$ coincides with the coordinate basis $\{\frac{\partial}{\partial x^i}\}$ at the single point p, the connection does *not* vanish with respect to the coordinate basis, even at p.

In order to understand how this can be the case even though torsion is a tensor and does not vanish, it's helpful to note that in an anholonomic basis, the torsion tensor isn't equal to the anti-symmetric part of the connection. Relative to an arbitrary tetrad, torsion is given by $T^i{}_{jk} = -2\Gamma^i{}_{[jk]} - C^i{}_{jk}$. Here the $C^i{}_{jk}$ are the structure functions. These vanish for holonomic tetrads, but not for anholonomic ones, hence torsion can be non-vanishing in an anholonomic basis even while the connection coefficients vanish.[33]

Now ask whether the non-coordinate frame $\{E_i\}$ is a candidate for representing a phenomenological reference frame. The answer is clearly no. Anholonomic frames have no associated coordinates, and therefore do not have anything like associated Fermi coordinates, even where they are normal on a curve or region. These frames cannot be associated with inertial frames in the usual way.

3.3. Minimal coupling and the dynamics of PGT

The mathematical considerations above establish certain features of theories with a Riemann -Cartan geometry. First, we note that in any such

[33]Many thanks to Jim Weatherall for helping me to get clear on this.

theory there will be two classes of geodesic, one associated with the metric and Levi-Civita connection, and another associated with the Riemann-Cartan connection. Second, we note that there are no Fermi coordinates for the Riemann-Cartan connection, so the obvious candidates for inertial coordinates associated with the connection aren't available. The symmetric part of the Riemann-Cartan connection, with the same geodesics, does have normal coordinates defined for it.

Suppose the fundamental structure of the world is well-described by one or another Poincaré gauge theory. What should we then say about inertial structure? Will it reflect the metric structure, or the Riemann-Cartan connection structure, or both? The answer to these questions depends on the dynamics of the theory, and in particular how the geometrical objects appear in the Lagrangian. In GR, inertial structure is associated with metric structure precisely because the strong equivalence principle ensures that all the non-gravitational forces minimally couple to the Levi-Civita connection. By analogy, we can expect behaviour of material systems, and hence the phenomenological geometry of the theory, to depend on the nature of the coupling of the other interactions to the connection or connections. At this point, the fact that we're talking about a family of theories makes things complicated; there are far too many possible Lagrangians in the PGT literature to make analysing each one a possibility. Instead, I'll survey the space of possible Lagrangians, and then draw some conclusions based on the kinds of coupling that are possible:

1. **Minimal coupling to the Riemann-Cartan connection**: One option is that only the R-C connection, and not its derivatives, appear in the Lagrangian. It should be noted that, in cases with torsion, the minimal coupling prescription isn't straightforward. Minimal coupling in the matter Lagrangian does not, for example, lead to a minimally coupled Dirac equation.[34]In this case, one of the following applies:

    (i) The dynamics are such that torsion is only relevant on certain scales and for certain interactions. On other scales, the coupling is such that the antisymmetric part of the connection plays no part in the dynamics. This kind of scenario is the most common in PGTs; there is an argument, for example, that torsion doesn't

[34]See [15, p.632-633].

couple to electro-magnetism because such a coupling would violate E-M's gauge invariance, nor to scalar matter fields because they have no directional component.[35] On this kind of set-up, the connection that E-M phenomena feel will be a symmetric one, usually the Levi-Civita connection because this makes recovering GR results straightforward. So in domains where scalar matter and EM fields dominate, things will look much like they do in GR. Torsion will only be probed by certain kinds of matter at certain scales.

(ii) The connection with torsion couples minimally to all kinds of matter, but there turns out to be a translation of the Lagrangian available that returns minimal coupling to a symmetric connection. In the translated dynamics, torsion plays no role, so the theory is a notational variant of one with a symmetric connection.

(iii) Torsion couples to the dynamics in such a way that it's relevant at all scales and in all domains, and can't be translated away. This kind of theory isn't empirically viable.

2. **Non-minimal coupling to the R-C connection**: It might also be the case that derivatives of the connection appear in the Lagrangian. In practice, these kinds of PGT are usually (but not always) disregarded because they involve introducing new fundamental constants, but we'll review the options anyway (I assume non-minimal coupling will complicate things in such a way that no back-translation to a symmetric connection will be possible and there will be no analog of (ii) above).

   (i) Both the non-minimal coupling and the torsion can be ignored in some domains. As a result, there are domains in which there's effective coupling to a symmetric connection.

   (ii) Non-minimal coupling or torsion are relevant in all domains. Again, this is ruled out empirically.

As they're ruled out empirically, I'll here disregard options 1.iii and 2.ii. Option 1.i is the one most usually considered in the PGT literature, so it's

[35] See [18] and [17].

worth going into in some detail. What should we say about inertial structure in this case? In the standard case, on scales dominated by electromagnetic interactions matter will generally behave just as it does in GR; on these scales there will be inertial structure of just the kind that we usually measure. However, it's worth noting that this kind of coupling comes about not because the symmetric connection plays a fundamental role in he theory, but rather because the coupling with the Riemann-Cartan connection is such that torsion plays no role in the electro-magnetic dynamics. Even if the symmetric connection effectively felt by such matter is the Levi-Civita connection, and thus reflects the metric structure, the way in which the metric comes to be instantiated by phenomenological geometry is not straightforward.

What of those kinds of matter that do couple to torsion in the 1.i scenario? Here, behaviour will presumably depend on the nature of the coupling even where that coupling is minimal; we can't get at some universal behaviour by moving to a set of coordinates in which the torsion vanishes and working from there. There seem to be two questions here. First, is there any 'thick' inertial structure associated with the R-C connection? The answer to this question must be 'no'; the lack of normal coordinates for non-symmetric connections rules thick inertial structure associated with the Riemann-Cartan connection.

However, those who prefer a thinner definition of inertial structure might think that there is a second relevant question: Will torsion-feeling 'free' bodies follow the geodesics associated with the R-C connection? But here we may run into trouble over which bodies to classify as free. Granted, the theory should predict the motion of bodies in any given formal circumstance, including in the absence of non-gravitational forces, but it won't necessarily tell us which bodies are free. Does gravity here count as a force? If we go with a standard interpretation of geometry, we might think not. But it's hard to classify the trajectories under torsion alone as force-free in the usual sense; note that torsion will play a role in the equations of motion as written in any reference frame we choose, and the way in which torsion plays a role in the equations will depend on the details of body under consideration; the universality guaranteed by the vanishing connection in GR is not available here.[36] One might, perhaps, wish to insist that it's the *symmetric* part of the

[36]Note here that even in GR the geodesic theorem requires consideration of the dynamics (see, for example [25]), so considerations of Riemann-Cartan geodesic motion will require yet more complex dynamical considerations.

R-C connection that determines the inertial structure here, and then consider any deviations due to torsion to be non-inertial motion caused by a force. However, if the non-gravitational interactions have coupled to torsion, they won't take their canonical forms in these frames, and thus identify forces relative to the Levi-Civita normal coordinates in the usual way.

Hence, in the kind of scenario described by 2.i, in as much as inertial structure exists, it won't reflect the fundamental connection in the theory. Thus the phenomenological geometry will not match the fundamental geometry. Moreover, it's not clear that there could be a phenomenological spacetime geometry associated with the fundamental geometry. In this case, the phenomenological geometry is emergent.

What of option 1.ii, where matter couples to a symmetric connection? Teleparallel gravity is a theory of just this kind, and I've argued elsewhere [22] that is should be considered an unenlightening reformulation of GR. More generally, if a theory can be written in such a form that all matter couples to a symmetric connection, torsion is playing no role in the theory, and can be eliminated. Such a theory admits of the same kind of interpretation as general relativity.

Option 2.i covers a vast array of options, and much will depend on the nature of the non-minimal coupling. It's hard to know what to say about inertial structure in the domains where matter couples non-minimally to torsion; suffice it to say that it's very hard to see how anything like inertial structure could arise here; the comments on 1.i apply, but are compounded by the non-minimal coupling. Nonetheless, if there are domains in which there's effective coupling to a symmetric connection, these domains will possess inertial structure and hence phenomenological geometry, even if this phenomenological geometry fails to reflect the fundamental structure of the theory.

Thus we see that any viable PGT involving torsion *must* involve effective coupling to some symmetric connection in certain domains and at certain scales. That is, the connection that certain kinds of matter, and thus certain kinds of measuring instrument, will feel isn't the one that's fundamental to the dynamics of the theory. Due to subtle details of how the connection couples to matter in the lagrangian, much of the universe will behave as if the connection is simply that of GR, even though this connection isn't the one that appears in the gravitational dynamics. As a result, empirical results restrict us to PGTs that involve inertial structure that doesn't reflect the fundamental geometry of the theory.

These theories are written on a manifold with an associated metric; the case here is therefore somewhat different from a case in which there's no fundamental object that even seems to be a candidate for representing spacetime. However, one of the fundamental geometrical objects, $\Gamma^{\rho}{}_{\mu\nu}$, defined on this metric fails completely to possess the kind of spatiotemporal significance that the Levi-Civita connection has in GR. And the object that does have the traditional kind of spatiotemporal significance (some symmetric connection, usually $\{^{\rho}_{\mu\nu}\}$), is not one of the fundamental objects of the theory.

Where the connection whose structure is read off by 'ordinary' (non-torsion probing) matter is the Levi-Civita connection, it is tempting to think that the metric, which *is* a fundamental object, is what represents spacetime. However, in such cases the metric comes by its (approximate, effective!) phenomenological significance in a very roundabout way. It seems a mere coincidence that the fundamental connection couples to certain kinds of matter in just such a way that they effectively feel the Levi-Civita connection. And in theories where the effective coupling is to some symmetric connection other than the Levi-Civita connection, no such route will be available; the metric will lose its connection to inertial structure altogether! It therefore seems right to say that in some (and perhaps all viable) cases of PGT, inertial structure is emergent, in the sense of non-fundamental. Thus the phenomenological geometry of the theory, as revealed by measuring instruments and our choice of reference frame, is an emergent phenomenon. Despite being superficially close to GR in their geometrical structure, Poincaré gauge theories may be thought of as theories positing an emergent spacetime.

4. Conclusions

My aims here have been twofold. First, I have stressed that phenomenological geometry in general relativity is exactly the *kind of thing* that could be thought of as non-fundamental. The strong equivalence principle, properly formulated, already makes the link between GR's precise geometrical structure and the geometry we measure an approximate one. No great violence is done to our conception of this empirical, physical geometry by the suggestion that GR itself might emerge from a theory of quantum gravity whose geometry is either radically different, or absent.

Second, I hope to have convinced the reader that the emergence of physical geometry, far from being a radical move that overturns central tenets of our philosophy of physics, is actually a very natural, and perhaps inevitable, consequence of the need to look beyond GR for a theory of quantum gravity. There is no doubt that Poincaré gauge theory is not the final story, but the arguments for needing torsion to describe matter with spin are quite powerful; it seems likely that torsion will crop up at some level of description whatever the fundamental theory. And viable theories with torsion coupling to spin look likely to be theories that posit emergent geometry.

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